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# Creating Images on Non-Traditional Grids for Digital Image Processing

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**Abstract.** Digital image processing and computer graphics are based on the underlying grid (e.g., defined by the computer screen). However, the square grid has some disadvantages, e.g., topological paradox. Therefore, some non-traditional grids could effectively be used in these fields. On the other hand, there are wide-spread imaging devices (e.g., smart phones) that provide very high resolution images on the square grid. In this work, we propose resampling techniques to allow to save images on non-traditional grids. This gives the possibility that one can choose a grid for processing the image that has some better symmetrical and/or topological properties than the traditional grid has.

*Keywords:* digital image processing, computer graphics, geometric modeling, nontraditional grids, image resampling, symmetry

2020 *Mathematics Subject Classification:* Primary 68U10, 68U05, 52C20; Secondary 05B45

## 1. Introduction

In this paper we will address one of the most important preparation processes before using non-traditional grids in, e.g., imaging sciences, namely, how to create and have images with various shape of pixels, i.e., on various grids.

The usual question could be stated: why not the square grid? The square grids has numerous issues such as the edge handling. While vertical and horizontal lines are continuous, completely diagonal edges experience information gaps due to pixel connections. This phenomenon results from the inhomogeneous neighborhood relations of the square grid, where the distance of immediate neighbors alternates between $d = 1$ (side neighbor) and $d = \sqrt{2}$ (diagonal neighbor). This inconsistency extends to the violation of the digital version of the Jordan Curve Theorem, creating a topological paradox for which one of the best known solutions is that a curve can only be considered 'closed' if the neighborhood definitions for the object and its background are inconsistently applied (e.g., 8-connectivity for the curve and 4-connectivity for the background, or *vice versa*). To address this and related problems without using special methods, it is suitable to use non-traditional grids. Some more details and arguments can be found, e.g, in [4, 19, 27]. Non-traditional grids are useful and can be used also for path planning, building maps with robots and pictorial representation of sensor data [1, 7].

In this paper, as an initial step, we address the problem how to have images on various grids.

## 2. Historical Notes

There were some resampling techniques proposed in the literature earlier, e.g., the authors of [8] recommended resampling the image by uniting two-two square pixels to make a triangular pixel (trixel). This technique can be seen in Figure 1, on the left.

In a very similar manner, based on a rectangular grid in a brick-wall like arrangement hexagonal resampling can be done, see, e.g., in [24] and in [11]. Here again, the resampling is done by uniting two neighbor pixels to one new pixel as it is shown in Figure 1, on the right.

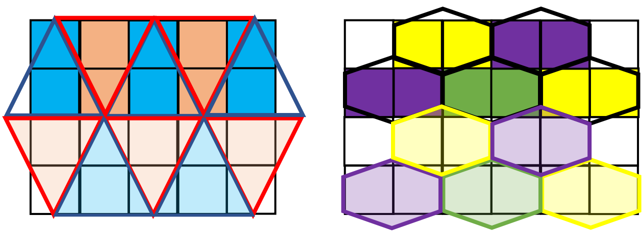


**Figure 1.** An idea for resampling images on the square grid for the triangular grid (left) and for the hexagonal grid (right) by uniting pixels.

One of the major problems of these low resolution resampling techniques that the resulting image is biased by the original image-grid. For example, the neighborhood system of the triangle pixels will inherit the square-grid based neighborhood and do not reflect the symmetry and the structure of the triangular grid. There are similar methods used, e.g., to make, test and reconstruct binary phantom images on the square grid and on the Cairo pattern, one of the best known pentagonal tilings [13].

In this paper, we consider the problem in a more general way. Our aim is to have a program tool that has a high-quality traditional uncompressed digital image (with standard square pixels) as input together with the targeted grid and the resolution that the resulting image should have, and the program is able to generate the resampled image on the defined grid with the specified resolution as output. The next sections show the details.

# 3. Proposed Grids

Our aim is to make a general program that has also as the part of the input the target grid and the resolution. Although our aim is to have the system general, in this section, we recall three different grids with various properties. These grids can be seen as motivations, on the one hand, for our 'preparation' task. On the other hand, with them we can also highlight what type of properties a grid may have and what type of problems one needs to deal with when preparing a software tool for resampling the images.

## 3.1. Hexagonal grid

The hexagonal grid is one of the best known and probably the second most used grid. It is among the three regular grids. The image is built up by regular hexagons (hexels). This image grid is a point lattice, i.e., it is closed under translations of any grid vector (i.e., vector connecting the center of any two hexels). The hexagonal pixels are more round than the square pixels, and the six neighbors are more isometrically oriented than the four side neighbors in the square grid. Because of the only one type of neighbor relation, the grid is relatively simple. Her introduced a symmetric coordinate system where 0-sum integer triplets are used to address the hexels [9]. There are no topological paradoxes in this grid. The grid gives the most efficient packing (of same size disks) in the plane, and also used in various places in the nature, e.g., honeybees build their honeycombs in this structure.

As this is the second best used planar grid, several image processing algorithms are already developed for images represented in this grid. Geometric transformations are investigated in [6, 10, 23]. Distance transform for hexagonal images is computed in [3]. Tomography and image reconstruction for images on this grid are considered, e.g., in [14, 15, 20] with various approaches. Cellular topology is discussed in [18, 26], just to mention a few results already available on this grid. There is also a textbook on hexagonal image processing [16]. Some advantages of this grid over the classical square grid are also highlighted in [4, 27]. However, maybe because our screens, projectors and other wide-spread hardware devices use mostly only the square grid, there are no well-known image storing formats and image databases (up to the knowledge of the authors) for such images. Since this grid is still very popular, we included it in our study.

## 3.2. Cairo pattern

The Cairo pattern is also well-known, it is named after paving some streets in Cairo. In fact, this is a pentagonal grid where the pentagons are identical, but not regular (see Figure 2). It is the dual tiling of the semi-regular tiling denoted by $(3, 3, 4, 3, 4)$ with the vertex configuration showing that at each vertex three triangles and two squares meet [5]. This grid was proposed to be used in image processing applications by G. Borgefors [2]. In the Cairo pattern there is a sole tile, but it appears in four different orientations. The Cairo pattern is involved in digital geometry and image processing: coordinate systems and various digital distance functions are defined and computed in [12, 21, 25]; binary tomography algorithm was presented in [13]. The symmetry of the grid includes both hexagonal type and rectangular type symmetries, as the two mentioned coordinate systems reflect these facts and this is one of the reasons why this grid could have various applications connected to imaging. This grid is an example for a grid when a pixel may have various orientations. This property is typical for the Catalan grids, i.e., for the dual grids of semi-regular grids [19, 22].

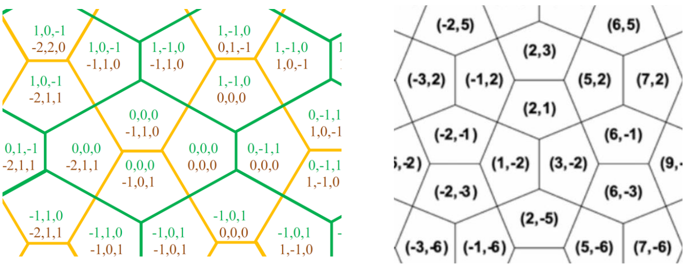



**Figure 2.** Cairo pattern with two coordinate systems from [12, 21], respectively.

## 3.3. Truncated nonagonal grid

The third grid we highlight here, contains non-regular enneagons (nonagons) and triangles. This grid was introduced in [17] as a possible solution to avoid topological paradoxes on a triangular grid. One may think about this grid that a type (one of the two orientations) of the triangles are pumped up in such a way that they can touch by edges the six closes same type pixels. Consequently the other types of triangles (the triangles with the opposite orientation) are squeezed not to have any extra corner neighbors, only the three side neighbor enneagons. Figure 3 shows a part of the grid. This grid is a typical example for grids built up by more than one type of pixels. All semi-regular grids also have this property [19].

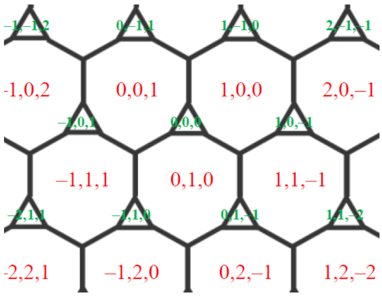


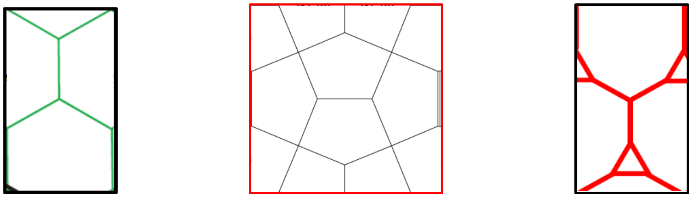

**Figure 3.** Truncated nonagonal grid with a coordinate system using integer triplets.

# 4. Resampling Process

Traditional resampling methods typically rely on a uniform, single-cell unit, such as a square or a simple hexagon. In contrast, we propose a methodology based on symmetric kernels composed of subdivided cells. This approach ensures that individual grid elements are not restricted to basic geometric primitives; instead, it allows for the construction of complex, heterogeneous grids, such as the truncated-nonagonal grid. By utilizing subdivided cells, we can maintain structural integrity even when multiple shape types are integrated into a single tiling system. Thus, for our method, the input is not only a high-quality uncompressed image on the traditional grid, but also a kernel describing the grid. This kernel must have a rectangular shape such that if one tiles the plane with its shifted copies in side-to-side manner, the kernel defines a periodic tiling of the plane. In Figure 4 we show the example kernels for our chosen grids. It is easy to observe how the hexagonal, the variously oriented pentagonal and the enneagon and triangle cells (pixels) tile the plain, respectively, based on the kernel.

The primary advantage of this kernel-based approach is the simplified workflow for establishing adjacency between current and preceding neighboring kernels. By defining the grid structure within a vertex matrix rather than as a singular repeating unit, we effectively eliminate spatial inconsistencies and the issue of unit shifting. This alignment is critical when mapping non-traditional grids onto the original image matrix, as it allows for the precise subdivision of kernel sizes while maintaining a perfect fit over the source data.

During the dynamic resampling process, grid units are synthesized from the divided components of both the current and the previous kernels, where the whole image contains $n \times m$ kernels and the current kernel is $k_c$, and the previous kernels are $k_c - 1, k_c - n$. Once the specific cell vertices are established, their exact



**(a)** Hexagonal grid kernel. **(b)** Cairo pattern kernel. **(c)** Truncated nonagonal grid kernel.

**Figure 4.** The kernels for the proposed grids.

coordinates are mapped onto the image matrix. The algorithm then identifies all pixels enclosed within the boundaries of a given cell and performs a color averaging operation. The resulting mean color value is assigned to the corresponding grid cell:

$$C_{cell} = \frac{1}{n}\sum_{i=1}^{n} P_i,$$

where $P_i$ is a pixel inside a grid cell. While this averaging process inherently results in a loss of high-frequency spatial and edge information, it is not a significant drawback in the context of our research as it is assumed that the input is an extremely high resolution image (i.e., it may contain a hundred megapixel, as such devices that may produce such digital images become more and more common in our everyday life). As the usual image processing tasks are working with lower-resolution images, our aim to output lower-resolution images on the given non-traditional grids. Thus, the framework is optimized for lower-resolution image processing where the primary goal is to maintain topological stability and prevent the geometric distortions typical of traditional square lattices.

# 5. Preparation for Custom Grids

The manual creation of unique kernels for non-traditional grids is a mathematically intensive and time-consuming task. To address this, we introduce an automated generation system supported by a dedicated graphical user interface (GUI). In this environment, users can define the fundamental vertex points of a kernel interactively. While these raw points do not constitute a complete grid definition on their own, their spatial coordinates serve as the foundational data for the automated construction of the vertex matrix.

Once the vertices are defined, the user partitions the regions for each cell or subdivided cell. This step is essential for establishing the neighborhood relations and connectivity rules that define the grid's topology. Automating this process ensures that the resulting grid adheres to the necessary symmetry and distance relations required for effective data handling.

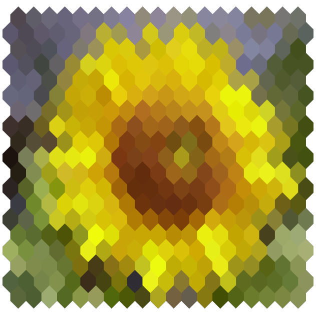
A key technical challenge lies in the development of a robust, automatic coordinate system for these custom structures. However, by enforcing a convexity constraint on the generated grid cells, we can streamline the rendering process. Convex cells allow for the implementation of automated triangle slicing (tessellation) algorithms, which are natively supported and highly optimized for the OpenGL API. This ensures that even high-resolution images can be resampled and rendered onto custom, non-traditional grids.

# 6. Some Experimental Results

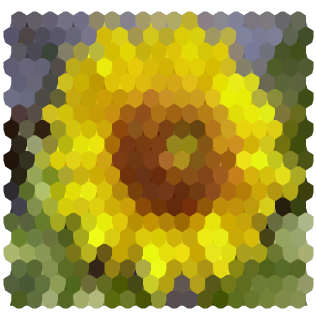
In this section, to show how our method works in the practice, we give some examples. One may notice that our method can also be used to create low(er) resolution images on the square grid, and thus, we may have a real comparison of the images on various grids. Later on these comparisons will be very important to

**(a)** Original sunflower image (resolution: $200 \times 200$).

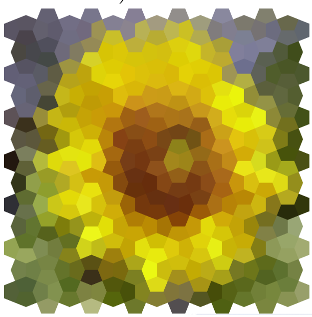
**(b)** Down sampled sunflower image (324 square pixels with $18 \times 18$ resolution).



**(c)** Hexagonal resampling result (370 hexels with around $19 \times 18$ resolution).



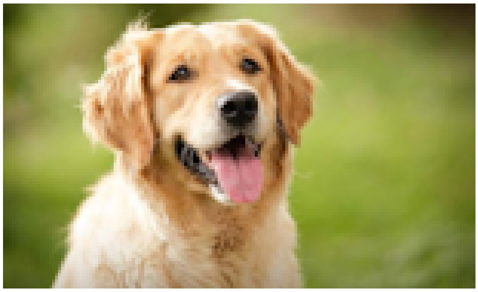
**(d)** Truncated nonagonal resampling result (370 nonagons and 371 triangles).



**(e)** Cairo pattern resampling result (364 pentagon).

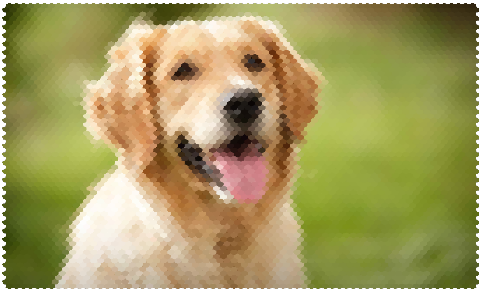
**Figure 5.** Results of various resampling processes: sunflower.

**(a)** Original image (710 × 430).



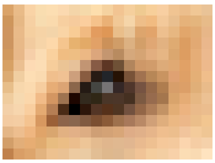
**(b)** Down sampled image (14322 square pixels with 154 × 93 resolution).

**(c)** Hexagonal resampling result (14220 hexels with around 119 × 118 resolution).

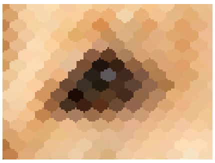


**(d)** Truncated nonagonal resampling result (4522 nonagons and 4523 triangles).

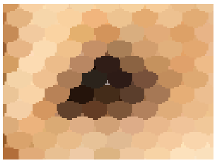
**(e)** Cairo pattern resampling result (14620 pentagons).



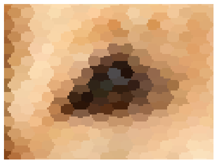
**(f)** Down sampled result (eye zoom).



**(g)** Hexagonal resampling result (eye zoom).



**(h)** Truncated nonagonal (eye zoom).



**(i)** Cairo pattern resampling (eye zoom).

**Figure 6.** Results of various resampling processes: dog and left eye zoom.

show possible advantages of image processing algorithms on non-traditional grids, in the practice. At this stage of research, one can only 'see' the differences of the images, as we can generate those. Thus, here some images and their resampled variants are shown in various resolutions, see Figures 5 and 6.

The resampled images are stored by having both the geometric data and image data of cells. As each grid contains only of finitely many types (and orientations) of their cells (pixels): we need to store the coordinates of the cell, its type (not applicable for the hexagonal grid, as it has a sole type) and its color value.

## 7. Conclusions and Future Work

In this paper we give some details on our project to use images based on non-traditional grids in digital image processing. The first, 'preparatory' step of the project was to able to create test images on various grid. We have developed a software tool that is able to resample (high-quality) images on any given non-traditional grid (defined by its kernel).

While the current work is preparatory, the objective is to develop a framework capable of integrating existing image processing algorithms and supporting the design of novel ones. The system enables users to define custom grid structures, providing a platform for testing and validation. By maintaining consistent coordinate systems, the implemented processing algorithms remain applicable across different grid configurations.

Although there is a naive approach to store the resampled images, it is generally not efficient in terms of storage places. For the square grid various image compression strategies are also used. The next steps of the research line are more grid dependent, i.e., to find efficient way(s) to store the resampled images based on the structure and symmetry of the underlying grid. This can be based on a nice-structured coordinate system for the grid. For some grids, as we have mentioned, such coordinate system exists. However for some others, we may try to develop a coordinate system based on the kernel design (see also the related paper [22]).

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