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Development of numerical understanding at preschool age

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Abstract. The emergence and development of numerical cognition are fundamental to the formation of mathematical thinking, particularly during the preschool years. At this stage, children progressively acquire an understanding of the semantic content of numbers, their correspondence with quantities, and the ordered, stable structure of the numeral sequence. These early competencies form the basis for later formal mathematical learning and are widely regarded as strong predictors of subsequent achievement in school mathematics. Given the foundational role of early numerical competencies, systematic investigation of number-related skills before school entry is of both theoretical and practical importance. The present study aims to map and characterise the number-related abilities of children aged 5–6 years, approximately six months before the onset of formal schooling. The sample comprised 131 children in the final year of kindergarten. Participating institutions were selected based on availability. Within each kindergarten, children from groups preparing to enter primary school were randomly selected to participate in the study.

During the study, a purpose-designed assessment instrument was administered to evaluate children's numerical competencies across six task sets. The instrument measured: (i) comparison of quantities, (ii) recognition of quantity conservation, (iii) understanding of numerical equivalence, and (iv) application of simple arithmetic operations. Quantitative data analysis was conducted using Microsoft Excel and RStudio. Descriptive results indicate

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that the majority of participants demonstrated proficiency in recognising basic quantitative relationships. However, greater variability and uncertainty were observed in tasks assessing conservation principles and addition. Inferential analyses revealed a small-to-moderate negative association between total test completion time and overall performance. Specifically, Pearson's correlation coefficient was ($r = -0.34$), (95% CI $[-0.48, -0.17]$)]), and Spearman's rank correlation was $\rho = -0.23$, (95% CI $[-0.39, -0.05]$]). Linear regression analysis further indicated a significant negative slope ($b = -0.175$) ($p < .001$), suggesting that longer test duration was associated with lower total scores. From a practical perspective, the findings support the applicability of the developed assessment tool as a diagnostic instrument in early childhood education. Its use may enable kindergarten teachers to identify and systematically support the development of number-related competencies before formal schooling begins.

Keywords: kindergarten, mathematics, numerical understanding, preschool age

2020 *Mathematics Subject Classification:* 97C70

1. Introduction

A central responsibility of the kindergarten teacher is to direct the child's attention to the mathematical structures inherent in everyday phenomena. This involves supporting children in identifying and articulating connections, relationships, similarities, and differences embedded in their immediate environment. As Perlai [31]. argues, the guiding principle of educational activity at this stage should not be the memorisation of isolated knowledge elements but the facilitation of discovery-based learning, enabling children to uncover relationships and construct understanding through personal experience. In early childhood, knowledge acquisition is grounded in direct interaction with the surrounding environment. Experiences become meaningful knowledge through reflection, interpretation, and cognitive integration. Accordingly, educational practice should provide opportunities for multifaceted exploration, application, and reinterpretation of new concepts, thereby fostering the development of structured mathematical understanding.

When organising mathematical activities in kindergarten, play must be the primary mode of engagement. Play-based contexts enable children to interpret and make sense of reality, while simultaneously supporting the development of the cognitive functions essential for later formal learning. In this sense, play is the fundamental form of learning in early childhood. Empirical evidence supports this pedagogical approach: Björklund et al. [5] demonstrated that active teacher guidance within play contexts significantly increased children's engagement with specific mathematical concepts and promoted deeper mathematical reasoning.

In designing learning activities, due consideration must be given to the child's interests, needs, and intrinsic curiosity. Educational experiences at this stage should be experiential and intrinsically motivating, ensuring that participation arises from engagement rather than obligation. Through direct experience, children develop not only specific competencies but also dispositions, attitudes, and modes of reasoning that shape their broader cognitive development. Experiential learning in early childhood encompasses the cultivation of creative practices, sustained active engagement, the strengthening of informal relational dynamics, and, most importantly, the consolidation of understanding through reflective interaction with concrete situations. The mathematical insights and experiences acquired in this way contribute substantially to the child's holistic development. The interplay between experiential activity and emerging mathematical structures is therefore of particular educational significance, as it supports both domain-specific knowledge acquisition and the development of general cognitive capacities.

2. Literature review

An essential component of preschool mathematics education is the development of so-called pre-numerical concepts, that is, structured familiarisation with the domain of numbers. This includes determining and comparing quantities, developing a meaningful understanding of the number sequence (beyond rote recitation), and performing elementary operations. These competencies form the conceptual foundation for later formal arithmetic. Children encounter and use numbers from an early age, and the acquisition of counting skills follows a broadly identifiable developmental progression. Typically, this progression includes: (i) sensitivity to quantitative differences; (ii) recognition of categories such as "many" and "few"; (iii) imitative counting following adult models; (iv) random or fragmented recitation of number words from memory; (v) rhythmic recitation of the number sequence; (vi) gradual internalisation of counting principles; and (vii) coordinated one-to-one correspondence between objects and number words. At a more advanced stage, the child can enumerate a finite set of objects and determine its cardinality [3]. From a developmental perspective, counting initially emerges in the context of verbal play and rhythmic language use. It only later becomes systematically connected to object-based correspondence and symbolic numerical representation [2]. This transition – from verbal sequence to principled enumeration – marks a critical step in the formation of coherent numerical understanding.

Lev Vygotsky's sociocultural theory underscores the central role of social interaction and language in the development of numerical concepts [38]. From early childhood, children encounter numbers in a variety of everyday contexts. For example, while shopping with their parents, they observe price tags featuring numbers and hear number-related expressions at the checkout. At this stage, numerical terms often function as lexical elements within the child's vocabulary rather than as outcomes of deliberate counting procedures. A child may know that they have two legs, hands and eyes, but only one nose, or be familiar with tales such as The Three Little Pigs, without yet possessing a principled understanding of enumeration. Such experiences nevertheless constitute important precursors to formal counting [17]. In early development, counting often takes the form of reciting number words in sequence, similar to the repetition of a rhyme. Children are attracted to the rhythm, predictability, and positive reinforcement associated with such verbal routines. Upon entering kindergarten, it becomes the educator's responsibility to connect these memorised sequences with meaningful quantitative content [40]. What initially resembles a verbal game gradually develops into structured mathematical activity through engagement with pre-counting schemes and fundamental operations, including part-whole relations, comparison, addition, and subtraction [15, 16]. Mechanical counting, in isolation, has limited conceptual value. A child may be capable of enumerating objects without understanding the cardinal significance of a number such as six. Conceptual understanding emerges only when the child can reliably associate number words and symbols with specific quantities and structured representations. In this sense, pre-numerical concepts constitute both the precursor and the foundation of later mathematical knowledge [4]. Although the development of mathematical concepts typically begins at the pre-primary level, instructional practices may at times result merely in the formation of fragmented procedural models rather than coherent conceptual structures [28]. Numerical understanding includes the ability to perceive and compare small quantities, to judge relative numerical magnitude (e.g., recognising that four is closer to three than to six), to internalise fundamental counting principles (such as the cardinality principle, according to which the final number word denotes the total quantity), and to construct and deconstruct sets through addition and subtraction (e.g., recognising that $3 + 2 = 5$ and $5 - 2 = 3$). It also entails understanding the linear ordering of numbers, whereby each successive number represents a quantity one greater than its predecessor and one less than its successor [36]. Accordingly, the systematic development of pre-numerical concepts within kindergarten education is central, as it lays the conceptual foundation for subsequent formal mathematical learning.

According to Richard Skemp, by the end of the preschool period, a child should be able to recognise numerals in various representational forms, including structured dot patterns such as those found on a standard dice [37]. Furthermore, the child should understand ordinal relations – for example, what it means to be fourth in a sequence – thereby demonstrating the ability to interpret numbers not only as labels but also as indicators of position. At this stage, children are also expected to comprehend fundamental relational concepts, including small–large, many–few, short–long, narrow–wide, empty–full, less–more, same, all, and none. However, the ability to produce correct answers when counting or performing simple procedures does not, in itself, demonstrate conceptual understanding. A child who arrives at a correct result by sequentially matching objects with fingers may be relying on procedural imitation rather than on an internalised concept of number. Such performance reflects operational competence at a surface level, but not necessarily a coherent grasp of cardinality or numerical structure. A crucial prerequisite for the development of pre-numerical concepts is the consolidation of the principle of the conservation of quantity. In pedagogical practice, this principle should be reinforced through varied and authentic real-life situations. For example, if seven apples are transferred from one container to another, the quantity remains unchanged, even if the spatial arrangement creates a perceptual illusion of an increase or decrease. Similarly, a set of ten toy cars retains its cardinality regardless of whether the objects are grouped closely together or dispersed across a larger area. Recognising that the number of elements in a finite set (particularly within the range below ten) remains invariant under changes in spatial configuration is a key indicator of emerging number sense. According to Jean Piaget, mastery of conservation typically develops around the age of six, following earlier stages in which children learn to distinguish equivalence and non-equivalence between sets. The acquisition of conservation thus represents a critical milestone in the transition from perceptually bound judgments to logically structured numerical reasoning.

Zsámboki states that the concept of number is fundamentally grounded in the relation of equivalence [41]. Its formation depends on recognising that distinct sets may share a common property – namely, identical cardinality – despite differences in spatial arrangement or perceptual configuration. The abstraction of this invariant property marks a decisive step in the development of numerical understanding. In practice, children may be able to compare quantities before acquiring a fully articulated concept of number. For instance, a child can determine which of two baskets contains more or fewer chestnuts without specifying the exact cardinal values involved. Through one-to-one correspondence (pairing), children can even compare relatively large sets without relying on counting procedures. Such strategies reflect an emerging sensitivity to quantitative relations that precedes formal numerical representation. Empirical findings reported by Mix et al. indicate that one-to-one matching facilitates preschool children's understanding of numerical equivalence [27]. Moreover, Mix emphasises that children often demonstrate competence in object-based numerical tasks before exhibiting comparable understanding in verbally mediated tasks [26]. This asymmetry suggests that early numerical cognition is grounded in perceptual and action-based representations, with subsequent verbal-symbolic abstraction.

Říčan [34]addresses the development of early mathematical abilities in preschool children, with particular emphasis on the emergence of counting and the conceptualisation of number. He argues that a preschool child is typically capable of counting to approximately ten, not merely mechanically, but with an emerging understanding of number as a representation of quantity. For example, when a child determines that there are three marbles, they may recognise that three denotes a quantity greater than one or two. At this stage, children are also generally able to classify objects by perceptual attributes such as colour or shape. However, Říčan cautions that although children can be readily trained to sort objects by size, colour, or shape, such instruction does not necessarily accelerate overall cognitive development. Moreover, the ability to recite the counting sequence up to ten does not guarantee an understanding of the correspondence between numerals and the cardinality of a set. Some children who can count from memory fail to recognise that number words denote the quantity of objects in a collection [14, 35]. The development of numerical understanding is closely intertwined with language acquisition. Numerals, linguistic structures, and language use support a more accurate representation of quantities and the development of counting skills [9]. Verbal counting may begin to emerge around age 2; however, its full development requires several years. Although children between the ages of 2 and 3 may attempt to count, their recitations of the number sequence are often unstable or inaccurate. With increasing age and experience, counting performance becomes progressively more reliable. Nevertheless, even at three to four years of age, children may not yet have consolidated specific number concepts (e.g., the concept of four). At this stage, accurate recitation of the counting sequence often relies on imitation and rote memory rather than on a principled understanding of counting procedures and cardinality [30]. Memorised counting sequences nevertheless play a foundational role in the development of numerical vocabulary and counting competence. Griffin [15] contends that, by approximately five years of age, children increasingly establish systematic connections between number words and quantities, thereby developing a more coherent "number sense" that supports the acquisition of basic arithmetic. By around six years of age, children are generally capable of accurately counting from one to ten and of coordinating number words with grouped objects, reflecting a more integrated understanding of counting principles [30].

In early childhood, engagement with mathematics does not originate in the formal manipulation of numbers; rather, it emerges from the child's attempts to relate properties to objects, persons, and phenomena in their immediate environment. Initially, numerical expressions are grounded in concrete experience. For example, the term "two" may not yet function as an abstract quantitative concept; instead, it is associated with embodied experience, such as holding two objects with two hands. Empirical observations indicate that children around the age of two can recite elements of the counting sequence (e.g., "one, two, three, four") while successively pointing to or touching individual objects [8]. However, even when a child can verbally enumerate numbers from one to ten, such recitation at this stage is typically mechanical. It does not imply an understanding of number as a representation of cardinality. The verbal production of number words precedes and develops independently of their quantitative interpretation. By approximately four years of age, children tend to develop a more coherent system of counting principles. As described by Cordes and Gelman [10], this system comprises several fundamental principles:

- The one-to-one correspondence principle: Each element of a set is assigned exactly one number word, and each number word corresponds to exactly one element.
- Stable order principle: The sequence of number words used in counting must follow a fixed and repeatable order.
- Cardinality principle: The number word assigned to the final element in a correctly executed count represents the total quantity of elements in the set.
- Abstraction (irrelevance) principle: Any collection of discrete entities – regardless of their natur – may be treated as countable objects.

The acquisition of these principles marks a qualitative shift from rote recitation of number words to an emerging conceptual understanding of counting as a structured and rule-governed activity.

For young children, counting and reciting number words are often enjoyable and playful activities. At first, however, these activities are largely devoid of quantitative meaning; number words function as elements of a memorised verbal sequence rather than as representations of cardinality. The child must first acquire a stable sequence of number words. The coordination of this sequence with objects develops gradually and requires sustained cognitive maturation [7]. In recognising the numerosity of a set, the child undergoes a significant conceptual shift. Specifically, the child must progressively inhibit attention to perceptual features of objects – such as colour, shape, material composition, or animacy – and instead attend exclusively to the invariant property of “how many” elements are present [6]. This abstraction from qualitative attributes to quantitative structure represents a critical step in the formation of number concepts.

From a pedagogical perspective, particular emphasis should be placed on the notion of equal quantity as a shared property of sets. The recognition that different collections may possess the same number of elements, irrespective of their qualitative differences, underlies the understanding of number as a measure of cardinality. Experiences involving comparisons of “more” and “less” should therefore be complemented by structured encounters with situations of equal quantity. Such experiences may be supported through multisensory activities, including motor tasks (e.g., performing the same number of jumps or steps as a model) and auditory patterns (e.g., clapping or vocalising a specified number of times). Activities involving “balancing”, understood as the addition or removal of elements to achieve equivalence between sets, are also central to this phase [41].

The development of a robust numerical concept presupposes an understanding of the equal cardinality of sets with different elements [37]. In formal terms, two sets have equal cardinality if there exists a bijection between them. When children are provided with appropriate concrete representations of one-to-one correspondences, counting serves as a constructive tool for establishing and consolidating this relationship. For the numerical concept to develop dynamically and to be grounded in an authentic sense of quantity, numbers must be encountered across a range of contexts and representations. If a child associates a number exclusively with a single fixed configuration, this may lead to rigid or incomplete understanding. Consequently, effective instruction requires systematic exposure to varied spatial arrangements and representational forms, thereby fostering flexibility and conceptual generalisation in early number learning.

The decomposition of numbers is another way to consolidate the emerging concept of numbers. In such activities, children partition a set into two or more disjoint subsets and observe that the total number of elements remains invariant under this transformation [41]. For example, when a cake is cut into several pieces and subsequently reassembled, its overall size remains unchanged. Analogous experiences with discrete objects enrich the child’s understanding of “equal quantity” and provide an experiential foundation for later mastery of basic arithmetic operations. Research by Mérei and Binét suggests that the number concept typically becomes established by the end of the sixth year of life [24]. Notably, this conceptual development often precedes explicit awareness of counting principles or formal terminology. The developmental process can be traced through observable changes in children's reasoning patterns, reflecting qualitative shifts in cognitive structure rather than the mere accumulation of procedural skills.

In a longitudinal study, Duncan et al. demonstrated that early mathematical competencies are stronger predictors of subsequent academic achievement than early reading skills, thereby underscoring the importance of timely and systematic support for the development of numerical understanding [11]. Although the role of digital technologies in early childhood education remains the subject of ongoing debate, empirical findings indicate their potential benefits. In particular, Räsänen et al. reported that digital learning environments – including interactive mathematical games – can effectively foster number sense and mitigate early mathematical difficulties, especially among socioeconomically disadvantaged children [33]. Additional studies corroborate the effectiveness of touchscreen-based applications in supporting the development of numerical concepts during early childhood [1, 19, 23, 25, 39].

In summary, the development of numerical understanding is a multifaceted process shaped by biological maturation, social interaction, language acquisition, and broader cultural factors. Therefore, the educator's role is to provide deliberate, developmentally appropriate support and create diverse opportunities that accommodate individual differences and promote the differentiated development of children's emerging mathematical competencies.

3. Materials and methods

3.1. Participants and sampling procedure

The empirical investigation was conducted in January 2025 and comprised a sample of 131 children from 26 kindergartens. The gender distribution of the participants is presented in Table 1. The study assessed the extent to which children expected to commence formal schooling within approximately 6 months had acquired selected pre-mathematical concepts.

Table 1. Gender distribution of the sample.

	Girls	Boys
Respondents	61	70
Percentage	46.56%	53.44%

The target population consisted of children aged 5–6 years, with an average age of 5.35 years. Development in numerical competencies during this period is both rapid and substantial. Previous research indicates that the level of numerical skill attained at this age constitutes a significant predictor of subsequent achievement in mathematics [18, 22].

A two-stage sampling procedure was employed. The design is described in detail below to ensure methodological transparency.

- Selection of kindergartens (institutional level). Participating institutions were recruited through convenience sampling. University students contacted kindergartens that were geographically accessible and willing to participate in the study. As this stage did not involve random selection, the institutional sample cannot be regarded as representative of the broader population. Consequently, the findings are not amenable to statistical generalisation beyond the participating institutions.

- Selection of children within kindergartens (individual level). Within each participating institution, children who were of school-entry age and scheduled to begin primary education in the given calendar year were first identified. From this subgroup, participants were selected using simple random sampling. Specifically, children's names were written on slips of paper and drawn at random. In most kindergartens, five children were selected; in some cases, six or seven participants were included, depending on the size of the eligible cohort.

In summary, while the selection of institutions was non-random, the selection of children within each participating kindergarten was fully randomised.

3.2. Measuring device

Basic numerical competencies were assessed using a self-developed measurement instrument designed to encompass the principal domains of the early number concept. The test comprises six units and fourteen tasks. These tasks address, inter alia, quantitative comparison through direct manipulation, comparison of sets presented in structured spatial arrangements, comparison of two sets by counting, understanding of counting principles, and elementary operational tasks involving partitioning and supplementing sets (i.e., decomposition and recomposition of quantities) [35].

The primary objective of the study was to evaluate the level of understanding of pre-numerical concepts among children aged 5–6 years. Based on the theoretical framework and the structure of the assessment instrument, the following research questions were formulated:

1. To what extent are 5- to 6-years-old children able to recognise and compare quantities presented in different spatial arrangements?

2. To what extent do children demonstrate an understanding of number constancy and equivalence across varying contexts?

3. What difficulties do children encounter when performing simple addition tasks using concrete objects or visual representations?

4. Is there a relationship between the time allocated for task completion and overall test performance?

These research questions guided both the design of the empirical investigation and the subsequent analysis of the collected data.

The tasks were constructed to address multiple components of the developing number concept, including quantitative comparison, equality, conservation of quantity, equivalence, counting competence, and elementary additive operations. The content and structure of the assessment instrument are summarised below.

1. Quantitative comparison by direct manipulation.
   The child is instructed to create two sets: one consisting of four apples and the other of ten apples. Within a narrative context, two hedgehogs (Francis and Mike) are said to have collected the apples. The child is asked to determine which hedgehog gathered more apples.
   Targeted concept: Comparison of cardinalities represented by concrete objects.

2. Comparison of sets in a structured spatial arrangement.
   Apples are arranged according to a predefined layout (e.g., grouped by colour). The child is asked to decide whether there are more red or green apples.
   Targeted concept: Visual comparison of quantities in structured configurations.

3. Comparison of two sets differing by one element.
   Four red apples and five green apples are placed in front of the child. The child is asked which group contains more items.
   Targeted concept: Sensitivity to small numerical differences and relative magnitude.

4. Counting and simple distribution tasks.

   a) Twenty apples are presented, and the child is asked to count the total number.
      Targeted concept: Mastery of the counting sequence and one-to-one correspondence.

   b) The child is informed that six children each receive one apple and is asked to take the appropriate number of apples from the bowl.
      Targeted concept: Application of counting in a distribution context (one-to-one allocation).

5. Conservation of quantity.
   All apples are first placed in a basket and then poured onto a plate in front of the child. The child is asked whether there are more apples in the basket or on the plate.
   Targeted concept: Understanding that quantity remains invariant under changes in spatial arrangement (conservation of number).

6. Dot-card tasks (symbolic and semi-symbolic representations).

   a–b) The child draws a card with a specified number of dots (e.g., three or six) and is asked to select another card displaying the same number of dots.
   Targeted concept: Recognition of numerical equivalence in pictorial representations.

   c–d) The child selects a card that matches either the number of dots or the colour of a given card (e.g., five or eight dots).
   Targeted concept: Differentiation between numerical and perceptual attributes.

   e) Two cards (e.g., red 2 and green 3) are presented, and the child is asked to determine the total number of dots.
   Targeted concept: Simple additive composition of quantities.

   f) The child is asked to select an additional card that results in a total of five dots (e.g., adding one to four).
   Targeted concept: Completion to a target number (additive complement).

   g) Two cards (e.g., red 7 and green 3) are presented, and the total number of dots is requested.
   Targeted concept: Addition within ten.

   h) Given a card (e.g., eight dots), the child is asked to select another card that results in a total of ten.
   Targeted concept: Decomposition and recomposition of ten (part–whole relationship).

3.3. Procedure

The assessment was administered individually, following standardised instructions. Children interacted with manipulable objects, including apples and dot cards. Tasks were presented in a predetermined sequence, and no assistance was provided during task completion; however, instructions could be repeated or clarified as needed. Both the accuracy of responses and the problem-solving process were systematically recorded.

Data collection was conducted in the participating kindergartens by final-year students in preschool education. Before testing, all students received training on procedural aspects of the assessment, including task explanation, observation protocols, and accurate recording of responses. Each child was tested individually to ensure that group dynamics did not influence performance. Given the attentional capacities of children aged 5–6, who typically sustain focus for approximately 10 minutes, the average test completion time was 13.57 minutes.

University students selected kindergartens for participation based on geographic accessibility, typically those near their residences. A minimum of five children from each selected kindergarten were included in the study. Within each institution, the pool of children scheduled to enter primary school in the current year was first identified, and participants were then randomly drawn from this group. Most kindergartens contributed five participants, while some provided six or seven pupils, depending on the size of the eligible cohort.

3.4. Scoring

Each test item was evaluated along three scoring dimensions, with the scale tailored to the specific task. Both dichotomous (0–1) and polytomous (0–2) scoring formats were employed:

- Correctness (dichotomous):
  - 0 = incorrect
  - 1 = correct
- Solution type (polytomous):
  - 0 = not solved
  - 1 = solved using a typical procedure
  - 2 = solved by inspection or non-standard reasoning
- Confidence (dichotomous):
  - 0 = confident solution
  - 1 = solution not delivered confidently

Alongside these quantitative scoring criteria, observations focused on the child's use of essential processes, such as quantity conservation, counting strategies, and overall estimation. In particular, the quality of counting procedures has proven to be a significant predictor of later arithmetic success [21, 32]. The total test score was computed as the sum of item scores; items were scored either dichotomously (0–1) or polytomously (0–2), depending on task structure.

3.5. Content validity

The tasks were designed to encompass the core domains of early numerical competencies, including quantity perception, comparison of sets, counting, decomposition and recomposition, and the conservation of number, in alignment with established international models and developmental learning trajectories for number concepts.

The dot card subtest specifically targets the child's ability to immediately recognise small sets (typically 1–4 elements) without resorting to counting, thereby assessing both subitising skills and the accurate mental representation of small numerosities. Comparative and decomposition tasks are employed to investigate part–whole relationships and the cognitive precursors of mental operations, providing insight into the child's emerging operational thinking [20].

The overall structure of the instrument aligns with foundational principles of counting, including one-to-one correspondence, stable order, and cardinality. The sequence of tasks follows a developmental learning trajectory: from quantity perception → counting → merging and subtracting → addition within ten. In particular, dot card tasks and additive tasks (sums to 5 and 10) are intended to capture the early emergence of mental counting strategies and the child's ability to manipulate numerical information cognitively rather than solely through concrete objects.

3.6. Criterion validity

The domains assessed – stability of counting procedures, constancy of number concepts, and decomposition and recomposition skills – have proved to be significant predictors of later mathematical achievement. Accordingly, the assessment instrument is well-suited to informing decisions on school readiness and guiding support for early numerical development [18, 22, 32].

3.7. Reliability

The internal reliability of the measurement instrument was evaluated using multiple estimates of Cronbach's alpha. Initially, the traditional Cronbach's alpha, based on Pearson correlations, indicated acceptable internal consistency ($\alpha = 0.77, 95\%$ CI [0.71, 0.83]). Corrected item-total correlations ranged from 0.21 to 0.63, suggesting that most items contributed adequately to the total scale variance, particularly the polytomous items in Block 6 (e.g., 6e, 6f). No single item, if removed, improved the overall alpha, indicating that the internal structure of the full item set is robust. Given that the scale comprises a mixture of dichotomous (0-1) and polytomous (0-2) items, reliability was further estimated using ordinal Cronbach's alpha, which is based on the polychoric correlation matrix and better suited for categorical data. This approach yielded a higher reliability coefficient $\alpha = 0.86$ (95% CI [0.83, 0.90]), reflecting excellent internal consistency. Under this model, corrected item-total correlations ranged from 0.22 to 0.75, with items 6e, 6f, 6h, and 4b identified as the most discriminating tasks. No "alpha-if-item-deleted" statistic suggested that removing any item would enhance overall reliability.

In summary, the instrument demonstrates a stable and coherent internal structure, and due to the categorical nature of the items, the ordinal alpha of 0.86 provides the most accurate estimate of the scale's reliability.

3.8. Ethical statements

The study was conducted in accordance with established ethical guidelines for psychological research and complied with the European legal framework governing research with children, as well as the principles articulated in the Declaration of Helsinki. All participants were provided with a detailed explanation of the study's objectives and the planned analytical procedures. To ensure anonymity, each child was assigned a unique identification code before the assessment materials were administered. Written informed consent was obtained from all parents or legal guardians before data collection, with explicit assurances that all study data would be kept confidential.

The Ethics Committee of Constantine the Philosopher University in Nitra received and evaluated the Request No. UKF/370/2025/191013:031 for a statement on the implementation of the research, including the accompanying documentation, on 1 October 2025. Following formal review, the Ethics Committee granted the application approval.

3.9. Results

The data were processed and analysed using Microsoft Excel and RStudio. Overall, children showed interest in the tasks. Table 2 presents the descriptive statistics for all fourteen tasks comprising the assessment instrument. In the initial three comparison tasks (Tasks 1–3), success rates approached ceiling levels, indicating that most children reliably recognised fundamental quantitative relations. In contrast, greater variability was observed in Tasks 4 and 5, particularly in Task 5 (conservation), where only approximately half of the participants produced correct responses. The results of the dot-card tasks reveal a well-structured developmental progression. Tasks permitting multiple correct solutions and polytomous scoring (Tasks 6b, 6c, 6e) yielded comparatively higher mean scores. Conversely, more complex tasks requiring mental integration and completion (Tasks 6f–6h) were associated with reduced performance. This performance gradient is consistent with established developmental models of numerical cognition and highlights the diagnostic sensitivity of the assessment instrument.

Children in this age group were occasionally distracted, consistent with typical developmental attention spans. They generally could not solve tasks by mere inspection; successful completion typically required counting.

Tasks 1–3: Quantitative comparison

The success rates for the first three tasks are shown in Figure 1. In Task 1, the teacher arranged apples into two piles and asked the children to identify which contained more. All but one child answered correctly. In Task 2, apples were presented in a structured pattern, and 6 children failed to identify the larger set,

Table 2. Summary of task performance (mean accuracy, standard deviation, sample size).

Task	Mean	SD	N
Task 1	0.992	0.09	131
Task 2	0.954	0.21	131
Task 3	0.939	0.24	131
Task 4a	0.580	0.50	131
Task 4b	0.794	0.41	131
Task 5	0.534	0.50	131
Task 6a	0.885	0.32	131
Task 6b	1.110	0.60	131
Task 6c	1.250	0.57	131
Task 6d	0.931	0.28	131
Task 6e	1.080	0.46	131
Task 6f	0.939	0.71	131
Task 6g	0.908	0.47	131
Task 6h	0.695	0.46	131

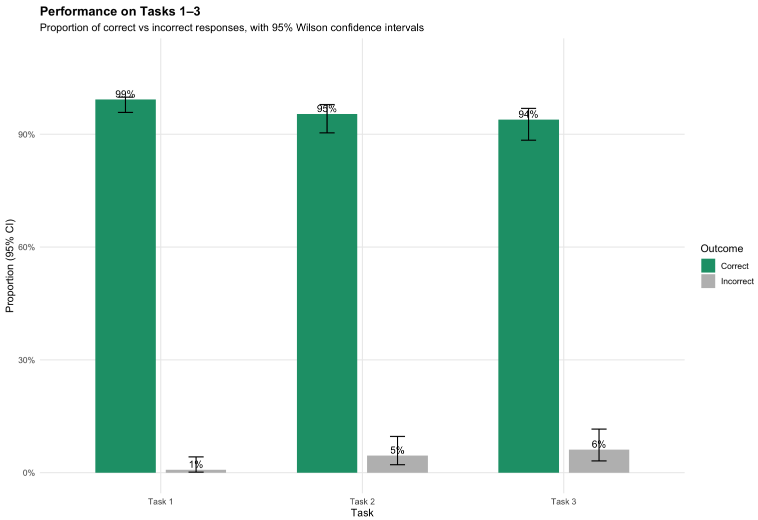

Figure 1. Success in solving tasks 1, 2 and 3.

while the remaining 125 completed the task successfully. Task 3 involved comparing two pre-set piles of apples by cardinality, and eight children were unable to solve it.

Task 4: Counting and distribution

Task 4 consisted of two parts. In Task 4a, children counted the apples in a pile, with many using their fingers, a typical strategy for this age group. Confidence in counting varied: only 55 children completed the task confidently, whereas 76 struggled (see Figure 2). Task 4b required distributing apples to feed six children, further assessing counting and understanding of quantity.

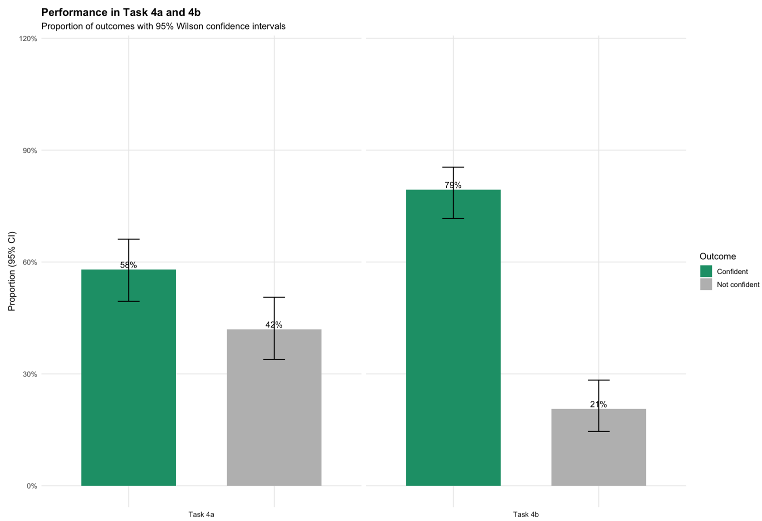

Figure 2. Success in solving task 4.

Task 5: Conservation of quantity
Task 5 assessed the child's understanding of equality and the constancy of quantity, regardless of spatial arrangement. Only 70 children solved the task correctly, while 61 failed, indicating that 46.56% of participants had not yet fully internalised the concept of constant quantity (see Figure 3).

Task 6: Dot card subtasks
Task 6 involved eight subtasks with dot cards of varying quantities and colours, designed to evaluate matching, addition, and the concept of "same number".

In the first two subtasks (Tasks 6a and 6b), a card with three dots was presented, followed by a card with six dots. Children were required to place another card showing the same number of dots as the second card. Initially, performance was recorded as correct or incorrect. Fifteen children failed Task 6a (see Figure 4). Subsequently, Task 6b assessed not only correctness but also the nature of the solution: whether the child employed a typical, age-appropriate strategy or solved the task by routine inspection. Seventeen children were unable to solve the task, 82 completed it using the typical strategy, and 32 solved it by inspection (see Figure 4, Task 6b). In Task 6c, children were asked to place a card showing five dots on top of a card that matched either the number of dots or the colour. Performance improved: only nine children failed, 80 solved it using the typical strategy, and 42 solved it by inspection (see Figure 4, Task 6c). Task 6d, which involved a card with eight dots and was otherwise analogous to Task 6c, demonstrated even stronger performance.

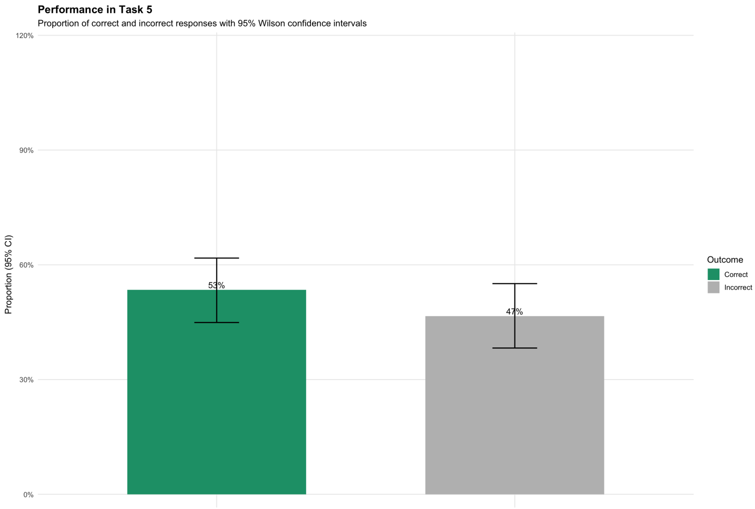

Figure 3. Success in solving task 5.

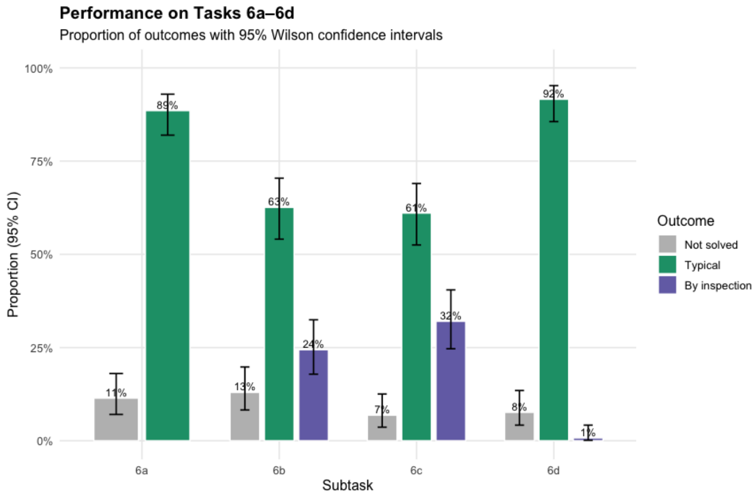

Figure 4. Success in solving tasks 6a–6d.

Only 10 children failed; 120 solved it using the expected age-appropriate strategy, and only 1 solved it by inspection (see Figure 4, Task 6d). In Task 6e, children were asked to match cards showing three and two dots on differently coloured cards. This proved relatively easy: only nine children failed, 103 solved it using the typical, age-appropriate strategy, and 19 solved it by inspection (see Figure 5, Task 6e). The subsequent subtasks were more challenging. In Task 6f, children received a red card with a certain number of dots and were asked to select another card so that the total number of dots equalled five. Thirty-seven children failed this task, 65 solved it using the typical strategy, and 29 relied on visual inspection (see Figure 5, Task 6f). In Task 6g, children were presented with two differently coloured cards, one showing seven dots and the other showing three, and were asked to determine the total number of dots. Performance improved slightly: 21 children failed, 101 solved the task using the expected strategy, and 9 solved it briefly by inspection (see Figure 5, Task 6g). The final subtask, Task 6h, required children to add a card to an existing eight-dot card to reach a total of ten dots. Forty children were unable to complete the task, while 91 solved it using the typical strategy; none relied solely on inspection (see Figure 5, Task 6h).

This pattern demonstrates that tasks requiring recognition of small quantities or simple matching were generally completed successfully, whereas subtasks demanding mental integration, addition, or conservation of quantity proved more difficult.

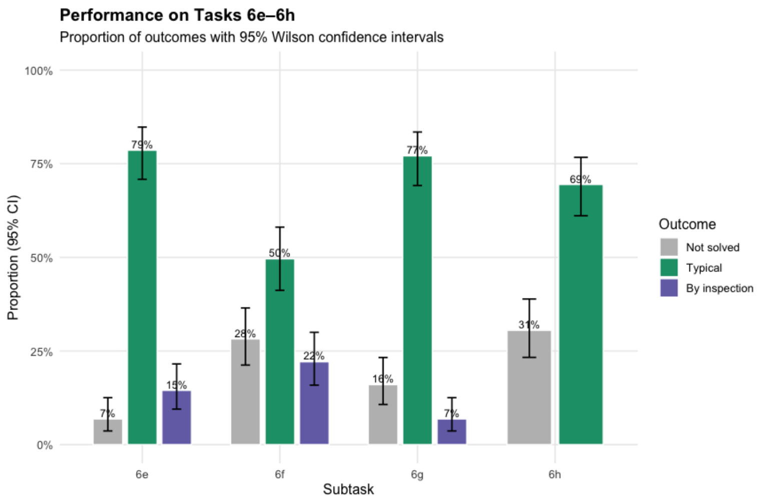

Figure 5. Success in solving tasks 6e–6h.

Regarding the relationship between task duration and performance, the scatterplot (see Figure 6). indicated a small-to-moderate negative association between task completion time and total score. Specifically, longer completion times tended to correspond to lower overall performance. Pearson's correlation confirmed a significant association: $r = -0.34$, 95% CI $[-0.48, -0.17]$, $t(129) = -4.06$, $p < 0.001$. Due to deviations from normality, Spearman's rank correlation was also calculated, yielding $\rho = -0.23$, 95% CI (bootstrap) $[-0.39, -0.05]$, $p = 0.008$.

A linear regression model, controlling for age and gender, further supported this relationship: the time slope remained significant ($b = -0.175$ score/minutes, $p < 0.001$; $R^2 = 0.147$). Overall, these results indicate that children who took longer to complete the tasks generally achieved slightly lower total scores, suggesting that slower task execution may reflect less stable numerical representations, uncertainty in counting, or increased cognitive load during task performance.

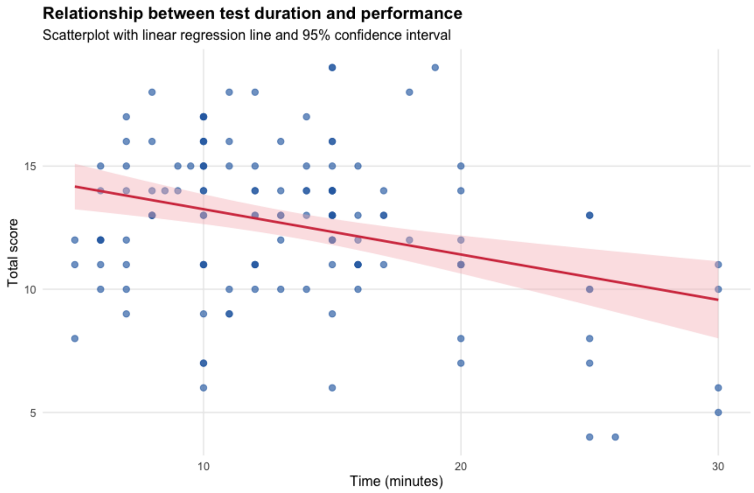

Figure 6. Relationship between test duration and performance.

3.10. Limitations

Cultural and social factors play a significant role in the development of numerical concepts. Variables such as socioeconomic background, parental support, and the availability of mathematical stimuli in the home environment can all influence a child's understanding of numbers and their meaning [25]. Although the present study did not examine these factors, they may have contributed to the variability in children's performance. Additionally, individual differences in cognitive and linguistic development, which were not assessed in this research, may also affect numerical understanding. Future studies should investigate these factors more systematically to provide a more comprehensive account of the complex influences shaping the development of early numerical competencies.

4. Discussion

The present study aimed to investigate the developmental level of number concept acquisition in preschool children approximately six months before school entry. As highlighted in the literature, the concept of number is multifaceted, comprising several interrelated components that rely on distinct cognitive skills. These components were systematically examined through the assessment instrument.

The first three tasks required children to compare the cardinality of sets and determine which contained more or fewer elements. Success rates exceeded 90% across all tasks, demonstrating that most children reliably recognise equivalence and non-equivalence between sets. However, many children counted items multiple times and showed uncertainty in applying counting procedures. Task 4 was subdivided into two parts, both of which proved relatively straightforward. In Task 4a, children counted apples, often using their fingers – a strategy consistent with developmental expectations at this age. These results align with national curriculum goals, which require children to count to ten by the end of preschool. Only three children counted consistently to twenty; most counted to five or ten, with occasional errors in sequence. Task 5 assessed the child's ability to perceive the equality and constancy of number, regardless of element placement. Nearly half of the children failed this task, reflecting both attentional lapses and an emerging but not yet fully established understanding of constant quantity. Task 6 involved eight subtasks, using cards with varying numbers of dots and multiple colours. Subtasks 6a and 6b required understanding the concept of "same number", a fundamental early mathematical concept. Fifteen children failed 6a, while 6b revealed that 24.4% solved it by visual inspection rather than applying a systematic strategy.

In 6c and 6d, children could match cards based on either the number of dots or colour; they prioritised dots. As dot quantities increased (e.g., eight dots in 6d), fewer than 1% relied on counting, yet overall success remained above 92%. Across these four subtasks, the mean success rate was 90.3%. Notably, in 6c and 6d, children often considered only the first member of the logical "or" operation, indicating partial understanding of multiple relational criteria. Addition tasks proved more challenging, particularly 6f and 6g, which required adding a card to an existing set to reach a target sum ($1 + \square = 5$ for 6f and $8 + \square = 10$). Performance declined relative to simpler tasks (e.g., 6e, $2 + 3 = \square$), with approximately 30% of children failing 6f and 6g. Difficulties were exacerbated by distracting dot sizes and jumbled arrangements, as well as reliance on one-by-one counting, reflecting that the concept of constant quantity is still developing. Task 6g, which required summing larger quantities ($7 + 3$), also showed a $\sim$10% decrease in success compared with simpler addition tasks.

A significant negative correlation was observed between task completion time and total score. Children who worked more slowly generally achieved lower scores, reflecting the influence of multiple cognitive factors. Consistent with prior research, processing speed, attentional capacity, and working memory are critical for early numerical performance, and individual differences in these domains are associated with an increased risk of later mathematical difficulties [12, 13, 29]. The negative association persisted in Spearman's rank correlation. It remained robust in regression analyses controlling for age and gender, indicating that slower task execution reflects a characteristic associated with numerical processing efficiency rather than being exclusively attributable to age-related maturation.

5. Conclusions

By the end of the preschool years, children typically show substantial progress in developing the number concept, which forms the foundation for later mathematical reasoning and numeracy skills in school. Ensuring that children reach an appropriate level of competence in this domain is crucial. Practical experiences, playful activities, and exposure to rich environmental stimuli play a central role in fostering early numerical understanding. Research indicates that most children can correctly sort numbers up to ten, with some extending this skill to twenty. They can count objects, assigning a number to each, and understand that the last number counted represents the total quantity. Children can also compare sets to determine which contains more, fewer, or an equal number of items. Early arithmetic concepts, such as addition and subtraction with concrete objects, begin to emerge during this period. Conservation of quantity is a key milestone in cognitive development. Although some 5–6-years-old children recognise that quantity remains constant regardless of arrangement, many are still influenced by the shape or layout of objects. The assessment instrument used in this study provides educators with a practical tool for identifying individual strengths and areas for improvement in the development of number concepts. Administering this assessment six months before school entry can guide targeted interventions to support each child's readiness for formal mathematical learning. Although the results support the internal coherence and developmental sensitivity of the instrument, its diagnostic use for school-readiness decisions should be considered preliminary, as criterion validity has not yet been established through external empirical measures.

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