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Perimeter defense game with nonzero capture radius in a circular target

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Abstract. In this paper, the problem of guarding a circular target wherein the Defender is constrained to move along its perimeter and has nonzero capture radius is posed and solved using a differential game theoretic approach. The Perimeter Defense Game is a special case of Pursuit-Evasion Game, where the goal of the pursuer is capturing the evader. In the Perimeter Defense Game the Attacker seeks to reach the perimeter of the circular target, whereas the Defender seek to align itself with the Attacker, thereby ending the game. The Defender has nonzero capture radius, which means that the Defender wins, when the distance between the Attacker and the Defender is smaller than the value of the capture radius. The Perimeter Defense Game can be divided into two cases: Win of Defender and the Win of Attacker scenarios. In the case when the Defender wins, the agents play a zero sum differential game, where the cost/payoff is the Attacker's terminal distance to the target. In the case when the Attacker wins, the agents play a zero-sum differential game, where the payoff/cost is the distance between the Defender and the Attacker. The analytic solutions of optimal strategies and the winning regions are also presented.

Keywords: differential game theory, Perimeter Defense Game, nonzero capture radius

AMS Subject Classification: 49N70 Differential games and control

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1. Introduction

In the reach avoid games there are two competitive teams, where one team attempts to arrive at a goal set in the state-space while avoiding some other undesired set of states. The goal of the opposing team is to prevent the first player from arriving at its goal [16]. The team consists of one or more players. The perimeter defense games are the special case of reach-avoid games. In a perimeter defense game the defender's team constrained to move along the convex perimeter and the attacker's team move with simple motion [10, 11]. The turret defense games are similar of perimeter defense games. In turret defense games the turret can be shoot the attacker from a certain distance. It can be transformated the game when the defender moves along a circular target and this way the problem can be solved easily using analytical methods [15]. The main difference between the perimeter defense and the transformated turret defense game is the winning condition of the attacker. During perimeter defense games, if the attacker reach the target at the same time as the defender makes an interception the defender wins, while in turret defense game the attacker emerge victorious.

The problem of guarding a target has many important application in real world. One example is protection of a building's perimeter against a sequentially arriving intruder [7]. In a real world there are not always information about the full state space, therefore, the information must be collected beforehand, for example with patrolling agents [12]. The target guarding can be applied in three dimension also, for example in the perimeter-defense game between aerial defender and ground intruder [6]. The survival is also a possible application, where one agent want to reach a safety zone while one [14] or more turret [4] want to neutralize its. Turret defense game with non-zero neutralization angle is also solved [8].

The perimeter defense game has many variant depending on the number of defenders, the number of intruders and the goal of the intruder(s). In a basic scenario there is one intruder and one or two defenders and the goal of the intruder is maximalizing the angular separation from the defender(s) when its reach the target [15]. The direct generalization is the perimeter defense game with more defenders and more intruders. This game can be solved with splitting into subgames with one or two defenders and one attacker [9, 10, 13].

In the reach avoid game the capture radius plays important role. The geometric solution of a target defense game where the attacker and also a defender can move freely in a full state space region with faster defenders with non-zero capture radius and a convex target area using the Hamilton-Jacobi-Isaacs equation is proven, but only partially appliable to our research, because of the constrained movements of the defender [3]. But in most of the existing paper discussing the perimeter defense games, the defender can capture with zero capture radius. There are proposals mentioned for the solution for the case of non-zero capture radius [12], but it is not complete proven and using only geometric solution. The most notable difference in this paper compared to the previous results that we studied the case when the capture radius is non-zero. In this analysis, we use the first order of necessary conditions for optimality according to a classical differential game approach [5]. The steps of solutions is identical to the basic turret defense game [15] with different termination and optimalization constraints.

The main contribution of this paper is the analytic solution with use of differential game approach of the Perimeter Defense Game with nonzero capture radius in a circular target. We give a step by step exploration of the solution.

The paper constructs as follows. After this introduction in Section 1, Section 2 presents the problem statement. Followed by Section 3, where the steps of the solution method are demonstrated. In section 4 the results are shown and the equilibrium flow field is presented. Last, in section 5 the conclusion and the future works with possible research directions are explored.

2. Problem statement

This paper formulates the target guarding problem wherein the Defender (D) constrained to move along the circular target perimeter and the Attacker (A) moves in the plane with simple motion. The Defender can make interception with r capture radius. In figure 1 can be seen the illustration and the rules of the game: R denotes the distance between the target center and the attacker, θ denotes the angular separation between the defender and the attacker and β the angular of the defender referred to the x axis. The goal of the Attacker is to enter the target, reach $\mathcal{T}$ without interception, and the goal of the Defender is preventing breach by the Attacker. Selected assumptions are made on the problem statement:

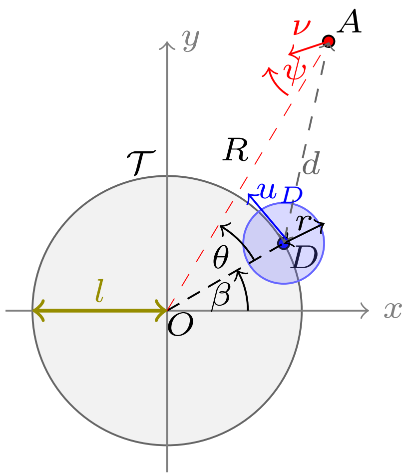

Figure 1. The illustration of the perimeter defense game with nonzero capture radius.

Assumption 2.1. *The target is a circle with $l = 1$ radius.*

Assumption 2.2. *The player's speeds are such that $0 < \nu \leq u_D = 1$, where ν is the speed of Attacker and u_D is the speed of Defender.*

Assumption 2.3. *The Defender makes interception with r capture radius. The $\mathcal{C}$ Capture Circle is defined as the set of the states of satisfying*

$$\mathcal{C} = \{(R, \theta) \,|\, r^2 \geq R^2 + 1 - 2R\cos\theta\} \tag{2.1}$$

Assumption 2.4. *The initial separation angle is such that $\theta(t_0) = \theta_0 \in [0,\, \pi)$*

Remark 2.5. The solution in case when $\theta_0 \in [-\pi, 0)$ can be determined from the symmetry result.

Assumption 2.6. *The initial Attacker distance is such that $R(t_0) > 1$, that is, A begins outside the target circle.*

Assumption 2.7. *The initial states are outside the Capture Circle*

$$(R_0, \theta_0) \notin \mathcal{C}. \tag{2.2}$$

The kinematics can be written as

$$f(\mathbf{x}, u, t) = \dot{\mathbf{x}} = \begin{bmatrix} \dot{R} \\ \dot{\theta} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\nu\cos\psi \\ \nu\frac{1}{R}\sin\psi - u_D \\ u_D \end{bmatrix} \tag{2.3}$$

The Defender control is the value of the speed of the defender and lies in the range $u_D \in [-1,\, 1]$ and the defender speed direction is always the tangent of the $\mathcal{T}$ target perimeter. The Attacker control is the heading angle referred to the line between the target center and the attacker and lies in the range $\psi \in [-\pi\,, \pi]$.

2.1. Defender wins scenario

In the Win of Defender (WoD) scenario, when D is able to make interception before A can reach the target, the agents play zero sum game over the cost-functional

$$J_d := \Phi_d(\mathbf{x}_f, t_f) = -R_f, \tag{2.4}$$

where the subscript f denotes the termination, Φ_d denotes the terminal value function that depends on the decision of the attacker and the defender. So the cost functional is the negative Attacker's distance from the target center at the end of the game. The Defender is the minimizing player and the Attacker is the maximizing player. The Value of the game if it exists, is the saddle-point equilibrium of the cost-functional over state-feedback strategies

$$V_d = \min_{u_D(\cdot)} \max_{\psi(\cdot)} J_d = \max_{\psi(\cdot)} \min_{u_D(\cdot)} J_d. \tag{2.5}$$

The terminal constraint is

$$\phi_d(\mathbf{x}_f, t_f) = \sqrt{R^2 + 1 - 2R\cos\theta} - r = d - r = 0, \tag{2.6}$$

that means that the game is terminated, if the distance between the Defender and the Attacker equals to the r capture radius. The final time t_f is the first time for which $d = r$. Thus, the Terminal Surface is defined as the set of states of satisfying (2.4)

$$\mathcal{J}_d = \{\mathbf{x} \,|\, R > 1 \quad \text{and} \quad d = r\}. \tag{2.7}$$

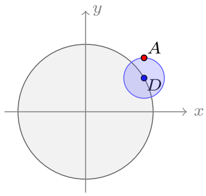

Figure 2. The illustration of the Defender wins scenario radius.

2.2. Attacker wins scenario

In Win of Attacker (WoA) scenario, when A is able to drive $R = 1$ while avoiding $d \leq r$, because of the separation angle is proportional the distance of the agents in case $R_f = 1$, the agents play zero sum game over the cost-functional:

$$J_a := \Phi_a(\mathbf{x}_f, t_f) = \theta_f - \theta_r, \tag{2.8}$$

where θ_r is the angular separation in the limiting case if $d_f = r$ and $R_f = 1$ it can be determined from theorem of cosines from AOD triangle in figure 1 $\theta_r = \arccos\left(\frac{2-r^2}{2}\right)$. The Defender is the minimizing player and the Attacker is the maximizing player. The Value of the game if it exists, is the saddle-point equilibrium of the cost-functional over state-feedback strategies

$$V_a = \min_{u_D(\cdot)} \max_{\psi(\cdot)} J_a = \max_{\psi(\cdot)} \min_{u_D(\cdot)} J_a. \tag{2.9}$$

Termination occurs when the Attacker reaches the target circle, therefore the termination constraint

$$\phi(\mathbf{x}_f, t_f) = R_f - 1 = 0. \tag{2.10}$$

The final time t_f is the first time for which $R(t) = 1$. Thus, the Terminal Surface is defined as the set of states of satisfying (2.10)

$$\mathcal{J}_a = \{\mathbf{x} \,|\, R = 1 \quad \text{and} \quad d \geq r\}. \tag{2.11}$$

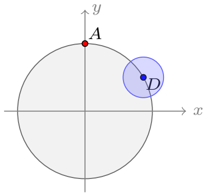

Figure 3. The illustration of the Attacker wins scenario.

3. Methods

The steps of the analytic solution of the Defender wins scenario (WoD) follow the steps of the Turret Defense Game [15], but we form the steps to our problem statement. We use the cost function (2.4) and the terminal constraint (2.6) during the derivation. The analysis is carried out according to a classical differential game approach [1, 5]. The solution of Attacker wins scenario (WoA) based upon showing satisfaction of the sufficient condition. The proposed equilibrium strategies of the Defender wins scenario and Value function substituting into the Hamilton-Jacobi-Isaacs equation [5].

3.1. Solution of Defender wins scenario

The steps of the following analytic solution follow the steps of the solution of zero-capture radius case [15] with subtituting the cost function (2.4) and terminal constraint (2.6).

The analysis is carried out according to a classical differential game approach [1, 5]. The Hamiltonian of the Defender wins scenario is

$$\mathcal{H}_d = -\sigma_R \nu \cos\psi + \sigma_\theta \left(\nu \frac{1}{R} \sin\psi - u_D \right) + \sigma_\beta u_D \tag{3.1}$$

where $\sigma \equiv [\sigma_R \quad \sigma_\theta \quad \sigma_\beta]^T$ is the adjoint vector. The Hamiltonian is a separable function of the controls u_D and ψ, and thus Isaacs' s condition [5] holds:

$$\min_{u_D(\cdot)} \max_{\psi(\cdot)} \mathcal{H}_d = \max_{\psi(\cdot)} \min_{u_D(\cdot)} \mathcal{H}_d \quad \forall \mathbf{x} \tag{3.2}$$

The Defender minimalize and the Attacker maximalize the Hamiltonian. The Defender control range is $u_D \in [-1, \quad 1]$ and the Attacker control range is $\psi \in [-\pi, \quad \pi]$. The equilibrium adjoint dynamics are given by

$$\dot{\sigma}_R = -\frac{\partial \mathcal{H}_d}{\partial R} = \sigma_\theta \nu \frac{1}{R^2} \sin\psi \tag{3.3}$$

$$\dot{\sigma}_\theta = -\frac{\partial \mathcal{H}_d}{\partial \theta} = 0 \tag{3.4}$$

$$\dot{\sigma}_\beta = -\frac{\partial \mathcal{H}_d}{\partial \beta} = 0 \tag{3.5}$$

The terminal adjoint values are obtained from the transversality condition [2]

$$\sigma(t_f) = \frac{\partial \Phi_d}{\partial \mathbf{x}_f} + \eta \frac{\partial \phi_d}{\partial \mathbf{x}_f} \tag{3.6}$$

$$\Rightarrow \begin{aligned} \sigma_{R_f} &= -1 + \tfrac{\eta}{r}(R_f - \cos\theta_f) \\ \sigma_{\theta_f} &= \tfrac{\eta}{r} R_f \sin\theta_f \\ \sigma_{\beta_f} &= 0 \end{aligned} \tag{3.7}$$

where η is an additional adjoint variable. Therefore, with (3.3), (3.7), the following hold

$$\sigma_\theta(t) = \frac{\eta}{r} R_f \sin\theta_f \quad \forall t \in [t_0\,,\ t_f] \tag{3.8}$$

$$\sigma_\beta(t) = 0 \quad \forall t \in [t_0\,,\ t_f] \tag{3.9}$$

Since $\sigma_\beta(t) = 0$ for all $t \in [t_0\,,\ t_f]$, the state component β has no effect on the equilibrium trajectory or the equilibrium control strategies. The terminal Hamiltonian satisfies [2]

$$\mathcal{H}_d(t_f) = -\frac{\partial \Phi_d}{\partial t_f} - \eta \frac{\partial \phi_d}{\partial t_f} = 0 \tag{3.10}$$

Since Φ_d and ϕ_d independent on time and $\frac{d\mathcal{H}_d}{dt} = 0$ so $\mathcal{H}_d(t) = 0$, $t \in [t_0, t_f]$.

The equilibrium control actions of the Attacker and Defender maximize and minimize (3.2), respectively:

$$\mathcal{H}_d^* = \max_{\psi} \min_{u_D} \mathcal{H}_d. \tag{3.11}$$

In order to maximize (3.2), the vector $[\cos\psi \quad \sin\psi]$ must be parallel to the vector $[-\sigma_R \quad \frac{\sigma_\theta}{R}]$. Therefore the optimal control of the Attacker can be expressed as:

$$\cos\psi^* = \frac{-\sigma_R}{\sqrt{\sigma_R^2 + \left(\frac{\sigma_\theta}{R}\right)^2}} \qquad \sin\psi^* = \frac{\frac{\sigma_\theta}{R}}{\sqrt{\sigma_R^2 + \left(\frac{\sigma_\theta}{R}\right)^2}}. \tag{3.12}$$

If $\sigma_\theta < 0$, this implies $\sin\psi^* < 0$ due to (3.12). However, this would mean the Attacker has a component of its motion that points towards the Defender due to Assumption 2.4. Thus, it must be the case that $\sigma_\theta > 0$ In order to minimize (3.1), the Defender's control must satisfy

$$u_D^* = \operatorname{sign}\sigma_\theta = 1, \tag{3.13}$$

since $\sigma_\theta > 0$.

To express the adjoint variable η must be substituting the equilibrium controls, (3.12) and (3.13), into the Hamiltonian (3.1) and evaluating at final time with (3.7) and (3.10) gives

$$\mathcal{H}_d^*(t_f) = -\sigma_{R_f}\nu\cos\psi^* + \sigma_\theta\left(\frac{\nu}{R_f}\sin\psi^* - u_D^*\right) \tag{3.14}$$

$$\longrightarrow \nu\sqrt{\sigma_{R_f}^2 + \left(\frac{\sigma_\theta}{R_f}\right)^2} - \sigma_\theta = 0 \tag{3.15}$$

An expression for σ_R is obtained by considering the Hamiltonian at a general time and substituting the equilibrium controls (3.12) (3.13) into the Hamiltonian (3.1):

$$\mathcal{H}_d^*(t) = 0 = -\sigma_R\nu\cos\psi^* + \sigma_\theta\left(\nu\frac{1}{R}\sin\psi^* - u_D^*\right) \tag{3.16}$$

$$\longrightarrow \nu\sqrt{\sigma_R^2 + \left(\frac{\sigma_\theta}{R}\right)^2} - \sigma_\theta = 0 \tag{3.17}$$

$$\longrightarrow \sigma_R = \sqrt{\frac{\sigma_\theta^2}{\nu^2}\left(1 - \frac{\nu^2}{R^2}\right)} \tag{3.18}$$

The ψ^* optimal heading angle can be determined by substituting the adjoint variables to the equilibrium the attackers controls (3.12)

$$\cos\psi^* = \frac{-\sigma_R}{\sqrt{\sigma_R^2 + \left(\frac{\sigma_\theta}{R}\right)^2}} = \frac{-\frac{\sigma_\theta}{\nu}\sqrt{1-\frac{\nu^2}{R^2}}}{\sqrt{\frac{\sigma_\theta^2}{\nu^2}\left(1-\frac{\nu^2}{R^2}\right) + \frac{\sigma_\theta^2}{R^2}}} = \sqrt{1 - \frac{\nu^2}{R^2}} \tag{3.19}$$

$$\sin\psi^* = \frac{\frac{\sigma_\theta}{R}}{\sqrt{\sigma_R^2 + \left(\frac{\sigma_\theta}{R}\right)^2}} = \frac{\nu}{R} \tag{3.20}$$

The equilibrium kinematics can be obtained by substituting the equilibrium controls (3.19) and (3.13) into (2.3) which yields

$$\dot{R}^* = -\nu\cos\psi^* = -\nu\sqrt{1 - \frac{\nu^2}{R^2}} \tag{3.21}$$

$$\dot{\theta}^* = \nu\frac{1}{R}\sin\psi^* - u_D^* = \frac{\nu^2}{R^2} - 1 \tag{3.22}$$

with the following boundary conditions $R_f > 1$, $r^2 = R_f^2 + 1 - 2R_f\cos\theta_f$.

Considering the differential equation obtained by dividing the equations in (3.21)

$$\frac{dR}{d\theta} = \frac{\nu}{\sqrt{1 - \frac{\nu^2}{R^2}}} \tag{3.23}$$

$$\Rightarrow \nu \left[\sqrt{\frac{R^2}{\nu^2} - 1} + \arcsin\left(\frac{\nu}{R}\right) \right]_{R_f}^{R} = \nu(\theta - \theta_f) \tag{3.24}$$

Define

$$g(R) = \sqrt{\frac{R^2}{\nu^2} - 1} + \arcsin\left(\frac{\nu}{R}\right) \tag{3.25}$$

$$\Rightarrow \nu(g(R) - g(R_f)) = \nu(\theta - \theta_f) \tag{3.26}$$

$$\Rightarrow \theta(R; R_f, \theta_f) = g(R) - g(R_f) + \theta_f, \quad r^2 = R_f^2 + 1 - 2R_f \cos\theta_f \tag{3.27}$$

Setting different θ_f, $0 \le \theta_f \le \theta_r = \arccos\left(\frac{2-r^2}{2}\right)$ in (3.26) describes equilibrium flow field for the Defender wins scenario. The equilibrium flow field gives the equilibrium trajectory from given terminal states. The optimal attacker path is the involute of a circle with radius ν.

The symmetric solution if $\theta < 0, t \in [0, t_f]$ can be solved in a same way. If $\theta_0 = \pi$ called dispersal surface, and in this case the positive and negative solution results the same value of the game. If $\theta_0 = 0$ called afferent surface and in this case the defender optimal trajectory is keep the zero angular separation.

The equilibrium state feedback control strategies for the Defender wins scenario are given by

$$\psi^* = \operatorname{sign}(\theta) \arcsin\left(\frac{\nu}{R}\right) \quad u_D^* = \operatorname{sign}(\theta) \tag{3.28}$$

The expression for ψ^* is obtained by (3.19) taking into account the sign of θ. Similarly, the Defender strategy is given by (3.13) taking into the sign of θ.

The Value of the game is

$$V_d(R, \theta) = -R_f \tag{3.29}$$

$$g(R_f) = g(R) + \theta_f - |\theta| \tag{3.30}$$

$$\Rightarrow V_d(R, \theta) = -g^{-1}(g(R) + \theta_f - |\theta|) \tag{3.31}$$

where g is defined in (3.25). Because V_d is defined using the inverse of the function g, it is necessary to show that $g(R)$ is monotic. Taking the derivate of (3.25) w.r.t. R gives

$$\frac{dg}{dR} = \frac{\sqrt{R^2 - \nu^2}}{\nu R}, \tag{3.32}$$

It must be that $0 < \nu < 1$ from Assumption 2.2 and from Assumption 2.6 it must be that $R > 1$ throughout the game. So we have $R > \nu$ and $R, \nu > 0$ which implies that $g(R)$ is monotonic.

The Value function does not have a closed form analytic expression since g^{-1} cannot be expressed in closed form.

The limiting case for the Defender wins scenario is one in which $R_f \longrightarrow 1$, $\theta_f = \theta_r$; thus the surface

$$\theta_{GoK}(R) = g(R) - g(1) + \theta_r \tag{3.33}$$

partitions the state space into regions of win for the Defender and Attacker, respectively,

$$\mathcal{R}_D = \{\mathbf{x} | \, |\theta| \leq \theta_{GoK}(R)\} \tag{3.34}$$

$$\mathcal{R}_A = \{\mathbf{x} | \, |\theta| > \theta_{GoK}(R)\}. \tag{3.35}$$

3.2. Solution of Attacker wins scenario

The solution of Attacker wins scenario based upon showing satisfaction of the sufficient condition for equilibrium via substitution of the proposed equilibrium strategies and Value function into the Hamilton-Jacobi-Isaacs equation [5]. The equilibrium state feedback strategies for the Attacker wins scenario is match those of the Defender wins scenario. The Value function is given by

$$V(R,\theta) = \theta_f - \theta_r = \theta - g(R) + g(1) - \theta_r \tag{3.36}$$

The Hamilton-Jacobi-Isaacs equation can be written as [5]

$$\min_{u_D} \max_{\psi} \left\{ l(\mathbf{x}, u_D, \psi, t) + \frac{\partial V}{\partial t} + V_x f(\mathbf{x}, u_D, \psi, t) \right\} = 0 \tag{3.37}$$

where V_x is the vector $\begin{bmatrix} \frac{\partial V}{\partial R} & \frac{\partial V}{\partial \theta} & \frac{\partial V}{\partial \beta} \end{bmatrix}^T$ and l represents an integral cost component. First, note that the cost, has no integral component, and thus $l = 0$. Also, the proposed Value function (3.36) is not an explicit function of time and thus $\frac{\partial V}{\partial t} = 0$. The vector V_x is obtained by differentiating (3.36) w. r. t. each state

$$V_x = \begin{bmatrix} \frac{-\sqrt{R^2 - \nu^2}}{R\nu} & 1 & 0 \end{bmatrix}. \tag{3.38}$$

The equilibrium dynamics f are given by (3.21). Substituting (3.38) , $\frac{\partial V}{\partial t} = 0$ and $l = 0$ into (3.37) gives

$$\frac{\partial V}{\partial R}\dot{R} + \frac{\partial V}{\partial \theta}\dot{\theta} = 0. \tag{3.39}$$

So the given Value function satisfies the Hamilton-Jacobi-Isaacs equation and this reason the equilibrium state feedback strategies are same of the cases Attacker and the Defender wins scenario The trajectories 3.26 are also same at the two cases, but in the Attacker wins scenario $r^2 < R_f^2 + 1 - 2R_f \cos\theta_f$ holds.

4. Results

In a Defender wins and Attacker wins scenario the agents have the same equilibrium strategies: the Attacker moves the tangent of the ν radius circle and the defender moves along the perimeter of the target towards the Attacker. The equilibrium flow field shows the trajectories in the (R, θ) plane for the Defender and Attacker wins scenario, also gives the terminal states and this way the winning regions. Figure 4

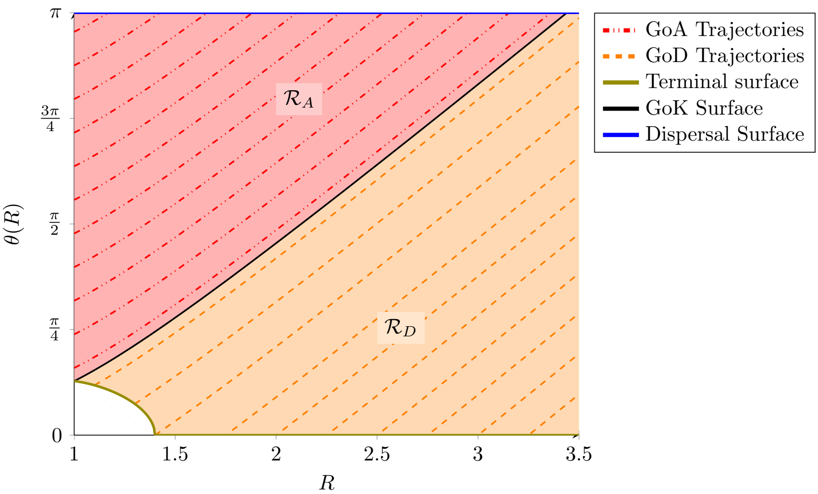

Figure 4. Full equilibrium flow field with $\nu = 0.8$ and $r = 0.4$.

shows the full equilibrium flow field in case $\nu = 0.8$, $r = 0.4$. The Attacker winning region and the trajectories denoted by red, the Defender winning region and the trajectories denoted by orange, the trajectory of limiting case denoted by black and the terminal surface of Defender wins scenario denoted by olive. The white region represents the $\mathcal{C}$ capture circle.

5. Conclusion and future works

In this paper we presented and solved the perimeter defense game with one Attacker and one Defender with r capture radius in a circular target. We created some assumptions and then showed the Defender and the Attacker win scenario and its solutions applying the Hamiltonian and the Hamilton-Jacobi-Isaacs equation ie. the first order necessary conditions for optimality and the sufficient condition for the equilibrium. The equilibrium state feedback strategies, the winning regions and the full equilibrium flow field are also presented. The equilibrium state feedback strategies are the same as the case of the point capture, but the winning regions and the equilibrium flow field depend on the r capture radius.

In future work we aim to solve the perimeter defense game with r capture radius applying more attackers and defenders. Perimeter defense game with general convex shape target, or general convex shape capture region is also a possible future work. It is also a possible generalization if $\nu > 1$, so the defenders have larger speed as the attackers, but there are more defenders than attackers. It is also possible future work when the Attacker(s) have penetration radius and this way they can reach the target earlier.

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