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Monitoring activities in prospective teachers' mathematics lessons

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Abstract. Research shows that teachers rarely or never pay attention to the "how" and "why" of using metacognitive skills. Mathematics teaching is more effective when metacognitive activities (planning, monitoring, and reflection) are implemented, and coherent discourses are developed in the classroom. The question arises as to whether the handling of these activities should be taught in a targeted way in teacher training or whether they are spontaneously integrated into practice by the end of the training. Our research investigated the emergence of monitoring activities in the lessons of prospective mathematics teachers in their secondary school teaching practice during classroom discussions. From the analysis of seven case studies, we conclude that monitoring activities are rare in the classroom despite the potential to trigger them is present. Prospective teachers cannot handle these situations appropriately spontaneously, so they need to be trained to deal with them.

Keywords: metacognition, monitoring, teacher training

AMS Subject Classification: 97C70, 97D40

1. Introduction

The starting point for our research is the experience that metacognitive activities are not emphasized enough in mathematics classroom discussions. The question arises whether the handling of these activities should be taught in a targeted way in teacher training or whether they can be incorporated spontaneously by the end of the training. Our research investigated the emergence of monitoring activities in the lessons of prospective mathematics teachers in their secondary school teaching

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practice during classroom discussions. We analysed the lessons of 7 prospective teachers from the point of view of how they managed situations that encouraged students to engage in monitoring activities in the lessons we highlighted. These classroom discussions followed the students' independent problem-solving and discussed the results, i.e., they focused primarily on the verification stage of problem-solving [19]. Our observations were not preceded by any targeted intervention or development in this area from the perspective of either the teacher candidates or the students. Our research investigated lessons in which students could search for and analyse errors. Lucangeli and colleagues [21] argue that error analysis can effectively promote the development of both metacognitive activities and mathematical performance.

2. Theoretical background

Schoenfeld [25] divided mathematical knowledge and behaviour in problem-solving into five interrelated components:

- Cognitive resources (knowledge base)
- Heuristic (problem-solving) strategies
- Monitoring and control (metacognition)
- Beliefs
- Practices

For our research, the monitoring and control (see later together as monitoring), or in a broader sense, the metacognitive component, will be highlighted and examined.

According to Flavell [9, 11], metacognition is an individual's knowledge of their cognitive processes and active monitoring, consistent regulation, and control of these processes. In most cases, the interpretation of metacognition presupposes at least a level of awareness that one should be able to report one's thoughts [6]. In this paper, we interpret such awareness in terms of metacognition.

After almost 20 years of work since the emergence of the concept, Flavell, Miller, and Miller [12] identified two main components of metacognition: metacognitive knowledge and metacognitive skills. Metacognitive knowledge refers to an individual's knowledge of their information processing abilities, the nature of cognitive tasks, and coping strategies for such tasks. Metacognitive activities refer to the regulative and executive skills (metacognitive skills) related to planning, monitoring, and reflecting on one's cognitive activities [22]. In this research, we investigate the monitoring activities of secondary school students from a specific perspective. Monitoring refers to the monitoring and controlling of cognitive activities and their outcomes [22].

As a metacognitive activity, monitoring occurs in all phases of problem-solving. It refers to continuously monitoring and evaluating one's cognitive activities to trigger regulatory processes [26]. Typically, monitoring activities are evaluated retrospectively in the final phases of the problem-solving models immediately after the cognitive task has been performed [13].

The relationship between metacognition and mathematical performance has been investigated in several studies. Among others, the 2003 PISA surveys, based on a sample of 1433 15-year-old students, showed that the relationship between an individual's level of metacognitive development and mathematical performance is significant [24]. All researchers report that metacognitive developmental level is one of the most important predictors of mathematical performance [7, 24, 32]. Verschaffel [31] concluded that metacognition is particularly important in mathematical problem-solving.

Several studies have shown a significant correlation between monitoring activities and mathematical performance in secondary school students [23, 28]. Incorrect monitoring can lead to deficits in activating relevant content knowledge and regulating cognitive processes [14]. Consequently, the quality of monitoring affects performance on a given task in the short term and the accumulation of cognitive and metacognitive knowledge relevant to mathematical problem-solving in the long term [20].

Depaepe and colleagues [7] have shown that teachers rarely or not at all address the "how" and "why" of using metacognitive activities. Dignath and Büttner [10] confirmed that teachers teach mainly cognitive and very few metacognitive strategies, suggesting that teachers may need training and explicit instruction on metacognition. Veenman and colleagues [30] found that teachers are willing to put effort into teaching metacognition within lessons but need the tools to implement metacognition as an integral part of their lessons. A study by Wafubwa and colleagues [33] investigated 213 Kenyan mathematics teachers' perceptions of metacognition and the impact of certain background factors on metacognition. An essential message of the study is that teachers can only teach students to be metacognitive if they are metacognitive themselves. It is necessary that teachers consciously use metacognitive activities and that their use is appropriately demonstrated and taught to students. The results pointed to the need to train teachers in implementing metacognition. Teachers mentioned large class sizes, heavy workloads, and lack of motivation as reasons for not using metacognitive activities. Teachers' use of metacognition was not influenced by teaching experience or educational qualifications.

Most researchers claim that metacognition can be learned and should be taught [3, 15, 21, 24, 27]. Studies have shown that mere observation or time spent on reflection is insufficient [8]. Metacognitive skills must be taught explicitly to develop mathematical skills, as they do not develop spontaneously from implicit exposure [1, 8]. One means of doing so is to expose learners to metacognitive-mathematical discourse while explicitly addressing the "how" and "why" of metacognitive activities. Cohors-Fresenborg and Kaune [4], and Kaune [16] have developed a mathematics curriculum that emphasized metacognitive activities and demonstrated the importance of a discourse-based learning culture for understanding mathematical problems. The learning process in the classroom can only lead to deep understanding when the associated metacognitive activities are elaborated, and coherent discussion and debate take place. Therefore, the class discussion should include discourse [5, 22].

- Discursivity refers to the activities carried out to support the coherence and accuracy of the discussion. Examples of discursive activities include the accurate (re)formulation and comparison of students' ideas, strategies, concepts, and misconceptions, as well as the linking of these to mathematical concepts or arguments.

- In contrast, negative discursivity refers to activities that negatively affect understanding meaning. Examples include self-answers or self-responsive questions, the use of inappropriate vocabulary or superficial and unclear sentences, the incorrect logical structure of an argument, and the introduction of an alternative idea into the discourse without reference to what has been said before.

The approach just mentioned is not new. In Hungarian mathematics education, similar ideas can be found among the methodological principles of Tamás Varga [29], such as shaping students' oral and written expression, developing independent opinion-making, and encouraging discussion. When students are given the opportunity to think independently in the classroom and to express their observations in their own words, the teacher is put in a situation where his or her cognitive load increases [18]. The cognitive load refers to the load in working memory (short-term memory) generated during the processing of information [2]. This is because the teacher has to immediately understand and evaluate unexpected situations and then decide how to deal with the situation that arises. The teacher can reduce this burden by making the lesson as predictable as possible, i.e., by adopting the students' expressions and suggestions, keeping the discussions between students in the background, and speaking and explaining rather than giving the lead. However, this differs from, or even contrary to, the methodological principles and approach mentioned above [18].

When these aspects are not taken into account in teacher training, it is no wonder that novice and in-service teachers opt for a teacher-dominated style of lesson management, with much less cognitive load, consisting of definitions and routine tasks, rather than a problem-focused approach to the subject matter, simply because it is instinctively less cognitive load [18].

3. Research questions

Q1. Do situations occur that can trigger monitoring activities in the lessons of teacher candidates during classroom discussions?

Q2. If yes, how do the teacher candidates manage these situations?

Q3. What factors might influence the occurrence of monitoring activities in classroom discussion during the teacher candidates' lessons in classroom discussions?

4. Method

Seven teacher candidates in their final year of training were asked to implement a lesson in a secondary school classroom during their coherent teaching practice. The lesson had to include at least one episode where students could actively participate in classroom discussions. The idea behind our request was that the teacher candidates should allow students to express their ideas and arguments. Teacher candidates were also asked to introduce a task that was solved incorrectly or to create a situation where a student's incorrect solution was analysed and discussed together. To meet our request, we suggested that teacher candidates present a problem solved incorrectly or create a situation where a student's incorrect solution is analyzed and discussed. We studied one lesson from each of the seven teacher candidates, focusing on such episodes.

Table 1 shows the identifies the grades of the classes, lesson themes, and episodes' themes with their duration time. The investigated episodes were not of equal duration. The duration of each episode was determined by the situation.

Table 1. Overview of the investigated episodes.

Prospective teacher	**Grade**	**Lesson theme**	**Episode**
A	9.	Proportionality	Discussing a faulty solution given by a group.
B	12.	Square root equations	Analysis of an incorrectly solved square root equation.
C	10.	Second-degree equations	Which of the two different solutions to the four short equations is correct?
D	9.	Solving word problems with equations	Which of the two different solutions is correct?
E	9.	Linear equations	Analysis of an incorrectly solved equation.
F	9.	Algebraic expressions	Which of the three algebraic expressions is the same as the original?
G	10.	Square root equations	Analysis of an incorrectly solved square root equation.

The lessons were audiotaped and transcribed. Furthermore, written notes were taken by an observer.

We used the categorisation system developed by Cohors-Fresenborg and Kaune (2007) to categorise classroom monitoring activities and discourses. Our coding system addresses the teacher and student questions/responses and the type of discursivity, as shown in Table 2 .

Table 2. The coding system.

Teacher asks	T?
Teacher answers	T!
Student asks	S?
Student answers	S!
Discursivity	D
Negative discursivity	ND

Dialogues are identified as units with the same number after the letter T or S. (For example, T1?, S1! is a dialogue unit, or S2?, T2?, S2! is another dialogue unit).

Manifestations that encourage or suggest monitoring activities are marked in red in the lesson.

In the presentation of the episode, some manifestations have been annotated to give a qualitative analysis of the situation regarding encouraging monitoring activities and involving students. The comments were motivated by the need to learn lessons and provide further advice, not by blaming the teacher candidates.

After coding, the following criteria were used to characterise the episodes:

- How many monitoring situations have been created in the episodes? How was the situation handled?
- How many discursive (D) and negative discursive (ND) manifestations did the teacher candidate have?
- How many Student-Student (S-S), Teacher-Student (T-S), Teacher- Teacher (T-T) dialogues occurred during the episode?

5. Results and discussion

5.1. A case study of a prospective teacher

The prospective teacher (denoted by A) was teaching a lesson to a 9th-grade class in an urban high school in 2021. The students have three mathematics lessons per week. The lesson theme was proportionality and percentages.

The problem statement that was the starting point for the subsequent discussion was the following.

Klári wants to paint the walls of her kitchen purple. The purple paint is mixed for her from three colours: blue, red, and yellow. The ratio of blue, red, and yellow in the mixture is $4:5:1$*. Six litres of blue,* 9 *litres of red, and* 2 *litres of yellow* *paint were found in the warehouse. What is the maximum number of litres of purple paint that can be mixed from the warehouse stocks?*

An expected solution applying the trial-and-error strategy is to check the ratio of the quantities of yellow, blue, and red paints using all the stored yellow, blue, and red paints, respectively. The answer is 6 litres of blue, $5 \cdot 1.5 = 7.5$ litres of red, and $1 \cdot 1.5 = 1.5$ litres of yellow paints, which give 15 litres of purple paint. Starting from the other two colours, we arrive at a contradiction.

Prospective teacher A formed groups of 4-5 students and asked them to solve the problem together and discuss the ideas if they had more than one. However, we observed groups in which only a few members discussed the problem, and the others worked independently. In the meantime, the teacher asked the groups if they needed help, how they were doing, how much time they needed, etc. After a while, he stopped waiting, and the class discussion started.

T: Good. Let's look at it, ... let's look at it together!

S: Teacher, I'm not sure we're going in the right direction ... – S1!

The situation initiated by the student may trigger monitoring activity (1). *The student's certainty about the correctness of his answer is weakened. This is an excellent opportunity for the teacher to initiate a discussion involving as many students as possible.*

T: How did you get started? – T1? **D**

The teacher opens a dialogue. *The question encourages monitoring activities, although this could have been preceded by asking the students what made them think they were not going in the right direction. This could have been a more valuable question from a metacognitive point of view because the students would have had to articulate the reason for the uncertainty.*

S: We started by writing 4, 5, and 1 in x and making an equation. – S1!

T: We know that the paint ratio is $4 : 5 : 1$. – T2! **D**

The student's mathematical language is problematic, and the teacher pointed out the related statement from the problem text to make it more transparent for the audience.

T: And what else do we know? How many litres of paint? – T2? ND

They started solving the problem again together, hoping the mistake would be found.

S: 6 litres blue, 9 litres red, and 2 litres yellow. – S2!

T: Good. We know these things. – T2!

S: Yes, we wrote these down so that $4x + 5x + x$ is $10x$, equal, and we added 6 to 9 and 2, which is 17. – S2!

The opportunity to trigger the monitoring activity is no longer available. A discussion and so a rich manifestation of monitoring activity have not emerged.

T: What is the problem with this? – T3? **D**

The situation initiated by the teacher may trigger monitoring activity (2). *Her question immediately suggested that the equation was wrong. She could have asked the students a less revealing question to get their opinions.*

T: The solution does not consider the proportions. Since you have poured all the paints together, they may not be in the same proportion ... – T3! ND

The teacher did not take advantage of the situation. She answered herself vaguely. He answered for himself, and not clearly. He could have tried a different question (e.g., What does this solution not consider?). She should have given the students more space to think.

S: 17. – S3!

Students did not understand the previous help.

T: You may not get these proportions right when you pour all the paints together. – T4! ND

The opportunity to trigger the monitoring activity is no longer available. *The teacher answered herself again instead of asking the students.*

T: So … you've come up with 1.7, which others have also whispered. – T4!

S: So, ours is not good for sure, teacher? – S5?

The situation initiated by the student may trigger monitoring activity (3). *The student did not understand what he had not considered in his solution.*

T: Let's have a look! If x is 1.7, then … – T5! **D**

The teacher opens a dialogue. *It was good that she did not give an immediate answer ("not good") but tried to help students see the contradiction.*

S: $6 \cdot 1.7$, and then we multiplied it, … but why … – S5!

The situation initiated by the student may trigger monitoring activity (4). *Why did the student multiply even by 6?*

T: Yes, and then you multiplied it also by 4, … – T5! ND

The teacher did not take advantage of the situation. *She quickly took control of the situation. She could have reacted to the "but why … " part of the student's sentence by asking the others to explain. From a metacognitive point of view, it was a valuable moment that could have been elaborated.*

S: … and by 5, then you've got how many litres of blue you need, … according to the ratios, yes, if you've got that, then you've got the red, which … – S5!

T: And did it come out? – T6? ND

The situation initiated by the teacher may trigger monitoring activity (5). *However, the question is too direct and suggestive. (Instead, e.g., What did you get? How do you evaluate the result?)*

S: We need 6.8 litres of blue, which is not good! – S6!

Now, it is clear that the solution is wrong. Although the student's answer refers to some monitoring activity, he did not have to add much work.

T: Yes, it is too much. Unfortunately, it's not good. Has anyone done it differently? – T6? **D**

The teacher did not take advantage of the situation. *It was good that the teacher did not explain the correct solution herself but asked the other students. Moreover, the lesson continued …*

The episode analysed dealt with a faulty solution that spontaneously emerged and had to be evaluated. One of the groups of students' solutions was discussed. At the end of the discussion, the students realised, with the help of the prospective teacher, that their solution needed improvement. The prospective teacher left another group of students the opportunity to explain the correct answer.

Table 3 shows that in the episode, we identified mainly teacher–student dialogues (T–S) and examples of the teacher answering her own question (T–T). No examples of student-to-student dialogue (S–S) were found.

Table 3. Types of the dialogues.

Type of the dialogue	Number
S–S	0
T–S	4
T–T	2

Table 4 presents that the prospective teacher's manifestations contained as many discursive as negative discursive elements.

Table 4. The quality of the discourse.

Quality of discourse	Number
discursivity	5
negative discursivity	5

In Table 5, we can see 5 situations to stimulate monitoring activities have emerged, 2 of which were initiated by the teacher and 3 by the student. In two situations, the prospective teacher successfully encouraged the student to engage in monitoring activities, but she did not take advantage of three situations.

Table 5. The number of promising situations related to monitoring activities.

	Initiated by the prospective teacher	Initiated by the student
Promising situations	2	3
Monitoring activity has occurred	1	1

From the metacognitive point of view, we can establish that the episode provided an excellent opportunity for monitoring activities. However, several of these metacognitively valuable situations were left unused. There were some self-answering questions or self-responses from the prospective teacher, which were classified as negative discursivity. Some of the teacher's prompts were too directed without giving space for students to formulate their ideas. Because the prospective teacher had robust control over the lesson, she missed several opportunities for more inclusive teaching. While she attempted to point out student mistakes, she could have involved more students in the discussion, creating a more conducive environment for dialogue.

5.2. Overall findings of the case studies of the seven prospective teachers

The chart below shows that the dialogues were mainly teacher-student dialogues, which is not surprising Figure 1. It should be added that many of these dialogues were characterised by long teacher utterances, or at least longer than those of the students. Students were often terse in their statements. The phenomenon of "talking to oneself" was observed in five of the seven teacher candidates. That is, when they asked a question, they answered it, or, after having expressed a unit of thought, they started a new one without letting the students reflect. The student-student dialogue was sporadic. An example of this was only in the lessons of two prospective teachers.

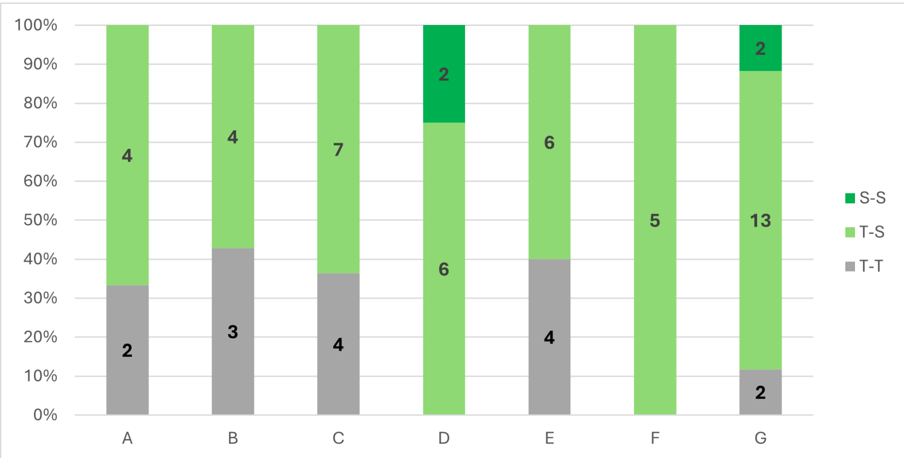

Figure 1. Types of dialogues in lessons.

Figure 2 shows the proportion of prospective teachers' manifestations identified as discursive (D) and negative discursive (ND). Discursivity, i.e., actions taken by the teacher to improve classroom communication, was the majority in the case of three prospective teachers (D, E, and G). The ratio of discursivity and negative discursivity was nearly the same for the two (A and F). For two teacher students (B and C), negative discursivity, i.e., unnecessary, misleading, or self-responsive contributions, was in most of the observed lessons. It should be highlighted that there was one prospective teacher in whom only discursivity and one in whom only negative discursivity was identified.

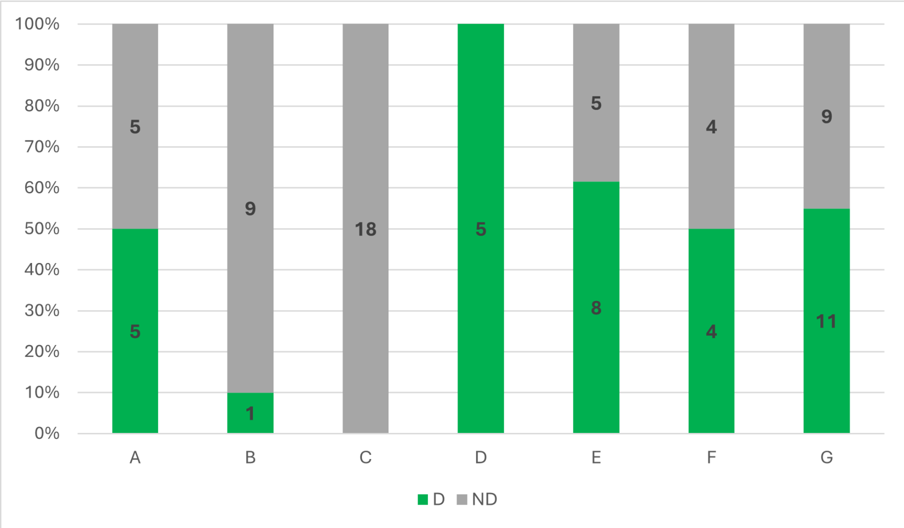

Figure 2. The rates of discursivity and negative discursivity.

Table 6 summarises the number of promising situations related to monitoring activities by prospective teachers. We found such situations implemented successfully for all prospective teachers except C. The relatively small number of such situations does not allow us to rank the prospective teachers; furthermore, the teaching episodes analysed were of different topics, durations, and intensities.

Table 6. Promising situations for monitoring activities in prospective teachers' lessons.

Prospective teacher	Promising situations	Monitoring activity has occurred
A	5	2
B	2	1
C	0	0
D	6	4
E	4	2
F	3	1
G	7	5

6. Summary and conclusions

The paper examined seven prospective teachers' lessons in seven different high schools. Teacher students were asked to create helpful discussions with students and were given the opportunity to share a faulty solution with the class. We aimed to observe how monitoring activities appear in classrooms under these conditions. Furthermore, we investigated the role of prospective teachers in encouraging monitoring activities [17].

We answer our research questions as follows.

Q1. Do situations occur that can trigger monitoring activities in the lessons of teacher candidates during classroom discussions?

Yes. Promising situations were observed that initiated by both prospective teachers and students.

Q2. How do prospective teachers manage these situations during classroom discussions?

Prospective teachers did not really engage the students or unfold the possibilities. They mostly did not have suitable questions to do so. In about half of the cases, they managed to get students to do monitoring activities, but the situation promised more than was achieved in all cases.

Q3. What factors might influence the occurrence of monitoring activities in classroom discussion during the teacher candidates' lessons in classroom discussions?

The prospective teacher's questioning culture, the degree of teacher guidance, the types of dialogue, and the students' activity can play decisive roles in the correct handling of metacognitively promising situations.

Overall, developing a meaningful discussion, a dynamic dialogue between students or teacher and students was very difficult or almost impossible. The tasks and the occurring situations held much more potential for engaging students in classroom discussion and encouraging monitoring activities than what materialised.

Our results support the findings discussed in the literature that teachers teach very few metacognitive strategies, suggesting that teachers may need training and explicit instruction [10] and tools to implement metacognition as an integral part of their lessons [30]. A study by Wafubwa and colleagues [33] reports similar experiences. Teachers can only teach students to be metacognitive if they themselves are metacognitive. It is necessary that teachers consciously use metacognitive activities and that their use is appropriately demonstrated and taught to students. Teachers should be trained in the implementation of metacognition in secondary schools.

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