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The effect of problem-based learning on students' learning outcomes*

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Abstract. Given the lack of consensus in the literature regarding the impact of problem-based learning on students' learning outcomes, we aimed to identify and understand the possible underlying factors that may contribute to the effectiveness of problem-based learning. To this end, we designed a series of lessons using a problem-based approach supplemented by heuristic strategies to investigate these factors. Two-cycle action research was implemented to explore lower secondary students' learning outcomes affected by problem-based learning and the purposeful use of heuristic strategies. We found that the combination of problem-based learning and the purposeful use of heuristic strategies positively impacts students' learning outcomes, and we explored this effect from both algebraic and geometric perspectives.

Keywords: problem-based learning, secondary school, learning outcomes

AMS Subject Classification: 97D40

1. Introduction

Problem-based learning (PBL) has beneficial effects on students' motivation, attitude [17], and critical thinking [2, 3]. However, the literature has no consensus on its impact on learning outcomes. While some studies claim that PBL increases student's achievement, other studies and analyses have either found an adverse effect or no significant increase in achievement [1, 8]. Further research reports the effectiveness of the purposeful use of heuristic strategies on learning outcomes [18].

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Based on these studies, we wanted to analyze the effect of PBL on students' learning outcomes, paying particular attention to the purposeful use of heuristic strategies.

Therefore, we (the author and two university experts) designed a two-cycle action research transforming two chapters (one algebraic, one geometric) from the curriculum. These chapters consisted of six, respectively, seven lessons, of which four were designed based on the PBL principles, including heuristic elements. We expected the combination of PBL and the purposeful use of heuristic strategies to impact students' learning outcomes positively. We aimed to explore this effect from both algebraic and geometric perspectives.

2. Theoretical background

Mathematics is about problems and solutions [11]. In mathematics education, a problem implies an obstacle that hinders achieving the goal. The way to overcome the obstacle is problem-solving and purposeful reasoning [16]. Heuristics have been generally recognized as a crucial component for problem-solving [14] because they are general suggestions on a strategy that is designed to help when we solve problems. A method that builds on problems and problem-solving is problem-based learning.

PBL dates to the 1950s and 1960s. Dewey [7] was perhaps the first to formulate the idea that knowledge should be imparted to learners in an active, exploratory way. In his work, Dewey advocates the introduction of active learning, whereby the teacher's task is not simply to make the students learn specific theories. Instead, the teacher's task is to create learning situations (problem situations) where students can acquire knowledge independently and help them manage them [6]. In this way, problem-based learning represented a paradigm shift from previous teaching-learning strategies.

Csíkos [5] defines PBL in mathematics as requiring students to analyze mathematical problem situations, to critically approach their own and their peers' minds, and they must learn to explain and justify their reasoning (see also [13]).

Note that hereafter, by the analysis of problem situations we mean not only problem-solving but problem-posing as well [15] (Figure 1).

A meta-analysis [8], which synthesized the results of 43 studies, sought to answer the following question: Do learners who learn using a problem-based approach achieve learning goals more effectively than learners who do not receive problem-based instruction? The research found an instant and lasting positive effect on learners' skills and abilities, while a negative effect was found in the area of knowledge. This analysis indicates that learners using the problem-based method have slightly less knowledge but, at the same time, remember more of the knowledge they have acquired. This mixed picture is confirmed by Hattie's study [12], which found that problem-based learning had no significant effect on student achievement. However, some researchers support the conclusion that problem-based methods improve the emotional domain of learners' learning, increase performance on complex tasks, and promote long-term retention of knowledge [1]. These studies and the

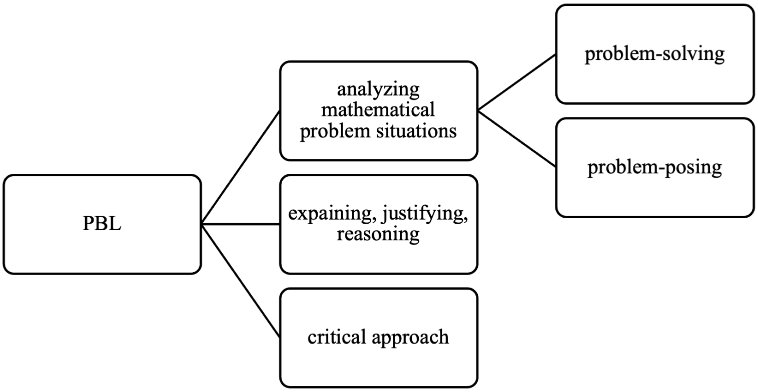

Figure 1. Definition of PBL.

contradicting results do not confirm the applicability of using problem-based learning to increase learning outcomes. However, the studies that have examined the impact of heuristic strategies on learning outcomes have all shown positive results.

Schoenfeld [18] used a control group study to show that all heuristics students improved from pre-test to post-test, while only one non-heuristics student made similar progress. In addition, heuristic students also had better persistence in problem-solving. A study by Singh et al. [19] also shows that the use of heuristics has a positive effect on the development of mathematical thinking of high school and university students, helping them to find ways to solve different problems through exploration.

Learning outcomes are closely related to the level of understanding. In the following subsection, we explore the different levels of understanding mathematics from two perspectives: algebra and geometry.

2.1. Levels of understanding mathematics

The Van Hiele levels [21–23] are regarded as a well-known model that suggest a possible way of structuring and describing people's understanding of geometry. They distinguish five levels of geometrical understanding (Table 1). According to this, a student advances sequentially from the initial to the highest level. Similar models were suggested for learning algebra as well; we considered the following six levels (Table 2) of algebraic thinking in primary and secondary education [10].

3. Research questions

Our research questions were formulated based on the previously presented research studies.

Table 1. Van Hiele levels [21–23].

Level	Description
0. vizualization	learners 'say what they see'
1. analysis	learners are aware of properties but do not reason based on them
2. abstraction	learners are able to recognize more formal properties and definitions
3. deduction	learners are able to use more formal reasoning, based on axioms, definitions and theorems
4. rigor	learners can argue precisely, comparing systems operating under different axioms and not being bound by the particularities of diagrams

Table 2. Levels of algebraic thinking [10].

Level	Description
Level 0:	learners can carry out operations with objects using natural, numerical, iconic, gestural languages
Level 1:	learners can use intensive objects (generic entities), the algebraic structure properties of N and the algebraic equality (equivalence)
Level 2:	learners can use symbolic–alphanumeric representations, although linked to the spatial, temporal, and contextual information; solving equations of the form $Ax \pm B = C$
Level 3:	learners use symbols analytically, without referring to contextual information
Level 4:	studying families of equations and functions using parameters and coefficients
Level 5:	analytical (syntactic) calculations are carried out involving one or more parameters

RQ1. How does the conscious use of heuristic strategies implemented in a problem-based approach affect students' learning outcomes?

RQ2. How do students reflect on their PBL process?

4. The setting of the study

The study was implemented with 61 students in two cycles. These students were 7th graders in the first cycle of the study and 8th graders in the second round, being part of the lower secondary school system in Transylvania, Romania. Among them, there were two students with special needs. The instruction language was Hungarian since Hungarian was the maternal language of the students. Action research has been implemented. Action research happens when people are involved in their research-practice to improve it and better understand their practice situations [9]. The action research reported here involves one mathematics teacher–researcher (the author) from Romania teaching 7th and 8th graders and two university experts in mathematics education. We selected one-one chapter from the 7th and 8th grade curriculum and aimed to design these chapters according to the curriculum, considering previous teaching– and research experiences. The title of the chapter from the first cycle was: Equations and problems that can be solved by equations of the form $ax + b = 0,\ a \neq 0$. The second cycle's chapter topic was 3-D shapes: cube, cuboid, cone, cylinder, prism, pyramid, etc. The structure of the two chapters is shown in the following table (Table 3).

Table 3. The structure of the chapters.

	7th grade	**8th grade**
Pretest	May 2021	November 2021
Lessons	**Topic of the lesson**	
	PBL: Equations	PBL: Cube and cuboid
	Practice (2)	PBL: Prisms
	PBL: Solving $ax + b = 0, a \neq 0$ type equations (3)	PBL: Pyramids
	Practice (4)	Practice (4)
	PBL: Solving word-problems by equations (5)	PBL: Cylinders and Cones
	PBL: Practice (6)	Practice (6)
		Practice (6)
Posttest	May 2021	December 2021
	Interview – Reflection	

As the table shows four problem-based lessons were implemented in both cycles. The main heuristic strategy we used in the process of designing problem-based lessons was pattern recognition. After the intervention, as described in Table 3, a semi-structured interview was conducted with 12 randomly selected students. The random selection was done after creating three categories of students: above-average, average, and below-average learning outcomes. The average was the mean of the mathematics grades of the two classes. Definition of the groups:

Group 1: students with below-average learning outcomes (mathematics grade 6 or below 6 in the previous semester[1])

[1] In Romania, grades are on a scale of 1-10, the lowest passing grade being a 5.

Group 2: students with average learning outcomes (mathematics grade 7 or 8 grades in the previous semester)

Group 3: students with above-average learning outcomes (mathematics grade 9 or 10 in the previous semester).

Two students per class were drawn from each category for the semi-structured interview. The categorization of the students is justified by Csapó's [4] study on the relationship between grades in mathematics and attitude towards mathematics.

5. Data collection

In this paper, we examine the results of the pre-and post-tests of the two cycles (4 tests in total), as well as the students' opinions gained from the interview after the intervention. The maximum score for the first cycle's test was 20 points, and the tasks from the tests were scored with the following in mind:

- 4 points – correct solution
- 3 points – sign error or minor calculation error (slight error caused by inattention)
- 2 points – the student gave an incorrect result, but the solution was initially correct (here we do not mean slight errors caused by inattention)
- 1 point – the student tried to solve the problem but made a mistake in the first half of the solution
- 0 points – the student did not write a solution to the problem.

The maximum score for the second cycle's test was 16 points, using the same point-giving system. An exception was made for two tasks that did not need explanation (one drawing, one multiple choice type question). In these cases, the maximum point was 2 points. The students' results in both cycles were analyzed based on the three groups described above (group 1: below average, group 2: average, group 3: above average). The analysis of these results is presented in the next chapter.

6. Quantitative results of the tests

First, we analyze the results of the tests from the first cycle. The Wilcoxon test shows a significant improvement for all three groups in (Table 4).

Although the p-significance value for the 3rd group is higher than for the other two groups, this result is also significant at the 0.05 level. This is due to the fact that 10 out of 17 students in this group (above-average students) already scored very high on the pretest (18/20-20/20). The students' averages are also plotted on a bar chart (Figure 2).

Table 4. The results of the tests in the 1st cycle.

Group	Wilcoxon test (W)	Wilcoxon test (z)	Wilcoxon test (p)	Effect size
1st group	8.000	−3.243	< 0.001	−0.895
2nd group	1.000	−3.351	< 0.001	−0.983
3rd group	0.000	−2.666	0.004	−1.000

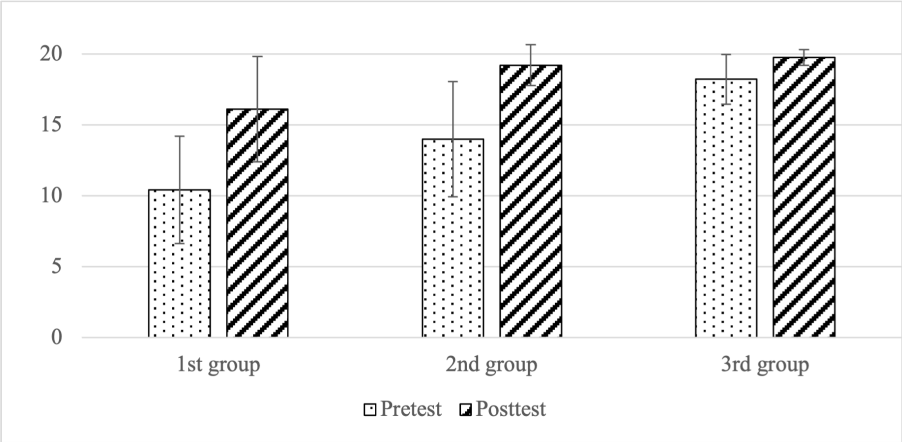

Figure 2. The results of tests from the first cycle.

The tests of the second cycle were analyzed similarly to the first one, with the following results (Table 5).

Table 5. The results of the tests in the 2nd cycle.

Group	Wilcoxon test (W)	Wilcoxon test (z)	Wilcoxon test (p)	Effect size
1st group	52.500	−0.801	0.217	−0.228
2nd group	10.000	−2.481	0.007	−0.780
3rd group	1.000	−3.110	0.001	−0.978

The results from the second cycle (Figure 3) show that although there was an improvement in the 1st group of students, it was not significant. However, in the 2nd and 3rd groups, learning outcomes improved significantly. The validity of this is made more significant in practical terms by the effect size.

The effect size for the Wilcoxon test is a measure of the difference's magnitude between two paired or matched samples. The value can range from -1 to 1, with values near 0 indicating that there is no effect and values near -1 or 1 indicating a

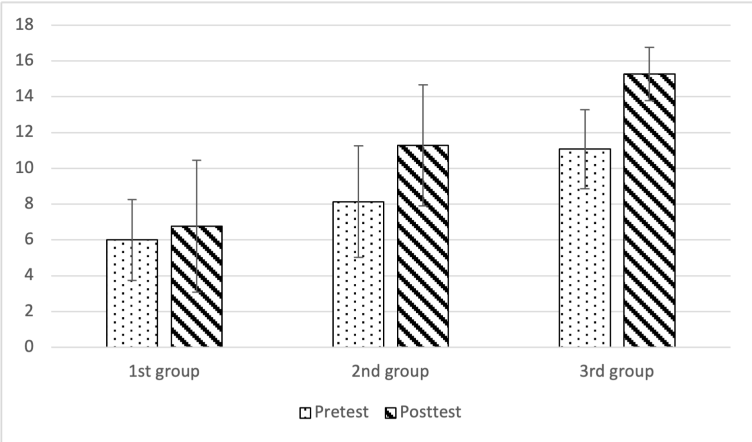

Figure 3. The results of tests from the second cycle.

strong effect. Although the analyses and effect size indicate a significant improvement overall, it cannot be said with certainty that this was due to the applied method. That is why we asked the students to share their experiences connected to these classes. The following sections consist of a qualitative analysis presenting the students' answers.

7. Qualitative analysis of students' answers

After having finished the teaching unit using a problem-based approach, we wanted to obtain the students' opinions, so we conducted a semi-structured interview with 12 students, as described earlier. In this section, we present why we are convinced that the significant increase in students' learning outcomes can be attributed to the intervention. We organized these reasons into six items by the motifs in the students' explanations:

1. the applied problem-solving activities can be used to develop logical thinking:

   S29: I like [. . .] that you have to think if I add another one, how it changes, and then you observe in a series how it changes and according to what equations, and this way, **you develop logical thinking.**

   S37: I **understood and had the logic in my head** how the results could come out.

2. the applied problem-solving activities allow exploring and expressing one's ideas:

   S53: [What I like is] that **you can discover something** that maybe **no one else has discovered**, and then **you're the first to notice** it, so to speak.

S13: What [I like] is that you can figure out the logic of how to do it, and then we usually **get the chance to figure out how to do it by ourselves**...

S32: I get a formula, or not necessarily a formula, just a rule that fits any of these numbers, I can apply it to any of these cases, **discover the relationship between them** and it feels good.

3. these activities can boost self-confidence and make the student feel good:

   S13: if I find out, then I'll **have a little confidence** so that I'm not such a lost cause after all. [Smiles] And **that makes me happier**...

   S32: I can [...] discover the relationship between them and **it feels good**.

   S17: In this class [...] **I felt confident** [...] **because**, if it's presented like that, I can **understand** it more easily than, [...] the exercises that are explained [...] if they say "expression", I'm like, "What?!" But if they tell me that it's the task with the toothpicks, then **I remember and understand how to solve it.**

4. the problem-posing activities require and develop creativity:

   S2: very, very good activities and I think **they also develop creativity**

   S54: I like it, it's good. **It's imaginative**, or how shall I say?

   S13: Well, **my creativity is not so unlimited**. [...] but if I have a starting point and I'm given what it's about, **I can figure it out**.

5. through problem-posing it is possible to have better insight into the structure of tasks:

   S28: [problem-posing] **allows me to see behind things** [...] how to solve them, in which case I'm the inventor, so **I set the boundaries** and do what I want with the task...

   S2: **you see the structure of the task, and how it is built**.

   S56: Well, I like to pose problems, because at least it's mine, and **I know that I put it together** and I know what it's about...

6. these kinds of activities help mathematical understanding:

   S2: [problem-posing] **contributes even more to our understanding**. And [...] we aren't solving only boring tasks, [...] they have **contributed to a better understanding.**

   S16: for example, if I have to solve a problem involving an equation, I may not understand it. However, if I have to formulate the equation or the text, **the whole thing becomes clearer and easier to understand.**

   S2: **helps you to understand** because you see the structure of the task, and how it is built.

   S29: I think it's a good idea to try to think a little bit backward [...] if you have a positive attitude, **it's helpful**...

> S28 Well, I think it's good if we can pose the problems, because **it will be much easier to understand** when we get such a task in the exam, and **it will be much clearer what to do**.
>
> S 58: [these activities] were good, because [...] **if you don't understand** it, and you **come up with an exercise** that's like the one you did in class, [...] **you might understand it better**.

Overall, we can say that students see problem-solving and -posing activities as logic developer activities, which also give them confidence. They find them challenging, but also creativity-boosting, which can promote better understanding. The perceived competence identified in students' opinions also supports the impact of the development.

8. Discussion

Considering the previously presented students' opinions we can claim that the significant increase in the learning outcomes must have a connection with the applied methods. This is supported by the perceived competence in students' answers. However, the results presented in the quantitative analysis show a symmetric structure in terms of the rate of development over the two cycles. Kruskal-Wallis test shows that the rate of development is significantly group dependent ($U(2) = 11.233, p = 0.004$). The reason why the 3rd group increased the least their learning results in the first cycle is apparent: their pretest scores were too high for that. The interesting question is how the 1st group managed to make a considerable improvement in the first cycle and barely improved in the second one. We assume that, although they were on the correspondent level in terms of content in the algebraic levels, they were not at the proper input level in the geometry chapter. The teaching process must start at the proper Van Hiele level to move from one level to the next. Moreover, if someone does not reach the expected entry-level, they will not be able to develop their understanding during the course [20]. This suggests that there could be several reasons why this group lacked in significant progress, for example: (1) the teacher did not design the chapter in such a way as to ensure development for the learners at the lower levels; (2) the material to be taught is too demanding for students of this ability, i.e., the curriculum is not based on age-appropriate Van Hiele levels for them. In any case, this means we need to explore the issue in more detail, which implies further research.

9. Conclusion

We designed a two-cycle action research to explore students' learning outcomes affected by PBL and the purposeful use of heuristic strategies. We applied a problem-based approach for two chapters (one algebraic, one geometric) from the curriculum. We found that the combination of PBL and conscious use of heuristic strategies positively impact students' learning outcomes, and we managed to explore this effect from both algebraic and geometric perspectives. The students who took part in the study were separated into three groups, taking into account their previous learning outcomes. We report a significant increase in students' learning outcomes, shown in both cycles of the experiment. However, while the 1st group of students (below-average) produced a significant increase in the first cycle (algebra topic), their development in the second cycle was not significant. We concluded that factors affecting the rate of development in terms of learning outcomes need further research. Although based on the quantitative analysis, we cannot conclude with absolute certainty that the significant increase is due to the method we used, the students' responses after the intervention indicate this assumption. We have summarized in six items the effects of the activities we used on students' perceived competence: problem-solving that involves pattern recognition can develop logical thinking (1), allows exploring (2) and positively impacts self-confidence (3). On the other hand, problem-posing activities develop creativity (4), give a better insight into the structure of the tasks (5) thus help mathematical understanding (6). Considering the PBL's impact on learning outcomes, we can claim that the purposeful use of heuristic strategies during PBL activities contributes to successful development. Also, the literature argues that PBL has many more beneficial effects, for example, on student's motivation, attitude, and critical thinking.

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References

[1] D. E. Allen, R. S. Donham, S. A. Bernhardt: *Problem-based learning*, New Directions for Teaching and Learning 2011.128 (2011), pp. 21–29, doi: 10.1002/tl.465.

[2] E. Báró: *Observing critical thinking during online pair work*, in: Critical Thinking Practices in Mathematics Education and Beyond, Rzeszów: Wydawnictwo Uniwersytetu Rzeszowskiego, 2022, pp. 128–136.

[3] E. Báró: *Teaching strategies for developing critical thinking skills*, in: Critical Thinking in Mathematics: Perspectives and Challenges, Rzeszów: Wydawnictwo Uniwersytetu Rzeszowskiego, 2022, pp. 128–136.

[4] B. Csapó: *A tantárgyakkal kapcsolatos attitűdök összefüggései*, MAGYAR PEDAGÓGIA 100.3 (2000), pp. 343–366.

[5] C. Csíkos: *Problémaalapú tanulás és matematikai nevelés*, Iskolakultúra 20.12 (2010), pp. 52–60, doi: 10.25656/01:7122.

[6] J. Dewey: *Democracy and education: An introduction to the philosophy of education*, Macmillan Publishing, 1916.

[7] J. Dewey: *How we think*, D C Heath, 1910, doi: 10.1037/10903-000.

[8] F. Dochy, M. Segers, P. Van den Bossche, D. Gijbels: *Effects of problem-based learning: a meta-analysis*, Learning and Instruction 13.5 (2003), pp. 533–568, issn: 0959-4752, doi: 10.1016/S0959-4752(02)00025-7.

[9] A. Feldman: *Editorial*, Educational Action Research 25.5 (2017), pp. 669–672, doi: 10.1080/09650792.2017.1388342.

[10] J. D. Godino, T. Neto, M. R. Wilhelmi, L. Aké, S. Etchegaray, A. Lasa: *Algebraic reasoning levels in primary and secondary education*, in: ed. by K. Krainer, N. Vondrová, Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education, 2015, pp. 426–432.

[11] P. R. Halmos: *The Heart of Mathematics*, The American Mathematical Monthly 87.7 (1980), pp. 519–524, doi: 10.1080/00029890.1980.11995081.

[12] J. Hattie: *The paradox of reducing class size and improving learning outcomes*, International Journal of Educational Research 43.6 (2005), pp. 387–425, doi: 10.1016/j.ijer.2006.07.002.

[13] E. Kónya, Z. Kovács: *Management of Problem Solving in a Classroom Context*, Center for Educational Policy Studies Journal (2021), doi: 10.26529/cepsj.895.

[14] R. E. Mayer: *Learning and Instruction*, Pearson Merrill Prentice Hall, 2003.

[15] E. Pehkonen: *Use of open-ended problems in mathematics classroom*, Dept. of Teacher Education, University of Helsinki, 1997.

[16] G. Polya: *On Learning, Teaching, and Learning Teaching*, The American Mathematical Monthly 70.6 (1963), pp. 605–619, (visited on 01/30/2024).

[17] L. Y. Sari, M. Adnan, H. Hadiyanto: *Enhancing Students' Active Involvement, Motivation and Learning Outcomes on Mathematical Problem Using Problem-Based Learning*, International Journal of Educational Dynamics (2019), url: https://api.semanticscholar.org/CorpusID:164775110.

[18] A. H. Schoenfeld: *Explicit Heuristic Training as a Variable in Problem-Solving Performance*, Journal for Research in Mathematics Education 10.3 (1979), pp. 173–187.

[19] P. Singh, S. H. Teoh, T. H. Cheong, N. S. Md Rasid, L. K. Kor, N. A. Md Nasir: *The Use of Problem-Solving Heuristics Approach in Enhancing STEM Students Development of Mathematical Thinking*, International Electronic Journal of Mathematics Education 13.3 (2018), doi: 10.12973/iejme/3921.

[20] Z. Usiskin: *Van Hiele Levels and Achievement in Secondary School Geometry*, Chicago: University of Chicago, 1982.

[21] P. M. Van Hiele: *Development and the learning process*, Acta Paedagogica Ultrajectina 17 (1959), pp. 1–31.

[22] P. M. Van Hiele: *Structure and Insight: A Theory of Mathematics Education*, Computer Science and Applied Mathematics, Academic Press, 1986.

[23] P. M. Van Hiele, G. Van Hiele: *A method of initiation into geometry at secondary schools*, in: Report on methods of initiation into geometry, ed. by H. Freudenthal, J. B. Wolters, 1958.