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# Two illustrating examples for comparison of uniform and proximal spaces using relators

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**Abstract.** We introduce (generalized) proximities in the same way as (generalized) uniformities in paper of Weil. We prove the equivalence of our new definitions with classical ones.

Using these analog definitions, we compare the properties of (generalized) proximities and (generalized) uniformities. The main parts of this paper are examples of an $(X, \mathcal{R})$ relator space such that $\mathcal{R}^{\#}$ is uniformly (and proximally) transitive, but neither $\mathcal{R}$ nor $\mathcal{R}^{\Phi}$ is proximally (or uniformly) transitive.

For this, we summarize the essential properties of relators, using their theory from earlier works of Á. Száz.

*Keywords:* (generalized) uniformities, (generalized) proximities, relators

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## 1. Introduction

At the beginning of the 20th century some mathematicians tried to define abstract topological structures. The most relevant results are Poincaré 1895, Fréchet 1906, Hausdorff 1914, and Kuratowski 1922.

Uniform spaces in terms of relations were introduced by Weil in 1937 [13].

Proximities were first investigated by Riesz in 1909 [9], Effremovič, and Smirnov in 1952 [2] and [10].

After the works of Davis, Pervin, and Nakano [1], [8], and [4] in 1987, Száz [11] introduced the notion of relator and relator space in the following way.

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**Definition 1.1.** A nonvoid family $\mathcal{R}$ of relations on a nonvoid set $X$ is called a relator on $X$, and the ordered pair $(X, \mathcal{R})$ is called a relator space.

In the last decades, a few authors investigated the interpretation of well-known topological properties in terms of relators. In 2021, Pataki introduced the general definition for (generalized) uniformities and (generalized) proximities in [5], moreover quasi-uniformities in [6].

For more details, see, for instance [5–7, 12], but for the readers' convenience, we summarize the necessary notions and notations.

**Remark 1.2.** With the usual notations, $\mathcal{R}$ is a relator on $X$ means that

$$X \neq \emptyset, \quad \emptyset \neq \mathcal{R} \subset \operatorname{Exp}(X^2),$$

where $\operatorname{Exp}(X)$ is the power set of $X$, and $X^2 = X \times X$.

If $R$ is a relation on $X$, $x \in X$, and $A \subset X$, then the sets

$$R(x) = \{y \in X : (x, y) \in R\}, \quad \text{and} \quad R[A] = \bigcup_{x \in A} R(x)$$

are called the images of $x$ and $A$ under $R$, respectively.

## 2. Preliminary concepts

**Definition 2.1.** If $R$ and $S$ are relations on $X$, then the composition of $R$ and $S$ can be defined, such that $(R \circ S)(x) = R[S(x)]$ for all $x \in X$.

Moreover, let $R^{-1} = \{(y, x) : (x, y) \in R\}$, $R^0 = \Delta_X = \{(x, x) : x \in X\}$ and $R^n = R \circ R^{n-1}$, for all $n = 1, 2, \dots$. Finally, we say that $R$ is

- reflexive if $R^0 \subset R$,
- symmetric if $R^{-1} \subset R$,
- transitive if $R^2 \subset R$.

**Lemma 2.2.** *If $R$ is a relation on $X$, and $A, B \subset X$, then*

$$R[A] \subset B \iff R^{-1}[X \setminus B] \subset X \setminus A.$$

**Definition 2.3.** If $\mathcal{R}$ is a relator on $X$, then the relators

$$\mathcal{R}^* = \{S \subset X^2 : \exists R \in \mathcal{R} : R \subset S\},$$

and

$$\mathcal{R}^\# = \{S \subset X^2 : \forall A \subset X : \exists R \in \mathcal{R} : R[A] \subset S[A]\},$$

are called the uniform and the proximal refinements of $\mathcal{R}$, respectively. For more details, see [7].

Moreover, for all $n = -1, 0, 1, 2, \dots$, we define

$$\mathcal{R}^n = \{R^n : R \in \mathcal{R}\}.$$

**Remark 2.4.** $*$ and $\#$ are really refinements as we defined in [7]. That is, they are self-increasing in the sense that

$$\mathcal{R} \subset \mathcal{S}^* \iff \mathcal{R}^* \subset \mathcal{S}^* \quad \text{and} \quad \mathcal{R} \subset \mathcal{S}^\# \iff \mathcal{R}^\# \subset \mathcal{S}^\#,$$

or equivalently, they are expansive, increasing, and idempotent, in the sense that

$$\mathcal{R} \subset \mathcal{R}^*, \ \mathcal{R} \subset \mathcal{S} \implies \mathcal{R}^* \subset \mathcal{S}^*, \ \mathcal{R}^{**} = \mathcal{R}^*$$

and

$$\mathcal{R} \subset \mathcal{R}^\#, \ \mathcal{R} \subset \mathcal{S} \implies \mathcal{R}^\# \subset \mathcal{S}^\#, \ \mathcal{R}^{\#\#} = \mathcal{R}^\#,$$

for all $\mathcal{R}$ and $\mathcal{S}$ relators on $X$.

Moreover, $\#$ is $*$-dominating, $*$-invariant, $*$-absorbing, and $*$-compatible, that is

$$\mathcal{R}^* \subset \mathcal{R}^\#, \quad \mathcal{R}^\# = \mathcal{R}^{\#*}, \quad \mathcal{R}^\# = \mathcal{R}^{*\#}, \quad \mathcal{R}^{\#*} = \mathcal{R}^{*\#}.$$

For all $n = -1, 0, 1, 2, \ldots$ the mapping $\mathcal{R} \mapsto \mathcal{R}^n$ of relators on $X$ is increasing.

Finally, $*$ and $\#$ are inversion-compatible, that is, for all $\mathcal{R}$ relators on $X$

$$\mathcal{R}^{*-1} = \mathcal{R}^{-1*} \quad \text{and} \quad \mathcal{R}^{\#-1} = \mathcal{R}^{-1\#}.$$

And we have that for all relators on $X$

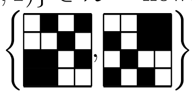
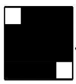
$$\mathcal{R}^{2*} = \mathcal{R}^{*2*}.$$

The following example shows that the analog assertion is not true for $\#$.

**Example 2.5.** Let $X = \{1, 2, 3, 4\}$, and

$$\mathcal{R} = \{\Delta_X \cup \{(1,2), (4,2), (2,1), (2,4)\}, \Delta_X \cup \{(1,3), (4,3), (3,1), (3,4)\}\}$$

is an elementwise reflexive and symmetric relator on $X$. Now, $\mathcal{R}^{\#2} \not\subset \mathcal{R}^{2\#}$, since $R = X^2 \setminus \{(1,4), (4,1)\} \in \mathcal{R}^{\#2}$ however $R \notin \mathcal{R}^{2\#}$.

Note, that $\mathcal{R} =$ , and $R =$ .

**Definition 2.6.** Let $\mathcal{R}$ be a relator on $X$, and $\square \in \{*, \#\}$ is a refinement for relators on $X$. We define the followings.

- $\mathcal{R}$ is $\square$-reflexive, if $\mathcal{R} \subset \mathcal{R}^{0\square}$;
- $\mathcal{R}$ is $\square$-symmetric, if $\mathcal{R} \subset \mathcal{R}^{-1\square}$;
- $\mathcal{R}$ is $\square$-transitive, if $\mathcal{R} \subset \mathcal{R}^{2\square}$;
- $\mathcal{R}$ is $\square$-fine, if $\mathcal{R} = \mathcal{R}^{\square}$.

For instance, we say that $\mathcal{R}$ is uniformly symmetric or proximally transitive instead of $*$-symmetric or $\#$-transitive.

Following Weil, we say that the relator $\mathcal{R}$ on $X$ is a generalized uniformity/generalized proximity on $X$, and the ordered pair $(X, \mathcal{R})$ is a generalized uniform space/generalized proximal space if it is

- uniformly/proximally reflexive;
- uniformly/proximally symmetric;
- uniformly/proximally transitive;
- uniformly/proximally fine.

Following the notations of [3], in [6] we introduced quasi-uniformities, and now, we define generalized quasi-uniformities/generalized quasi-proximities.

We say that the relator $\mathcal{R}$ on $X$ is a generalized quasi-uniformity/generalized quasi-proximity on $X$, and the ordered pair $(X, \mathcal{R})$ is a generalized quasi-uniform space/generalized quasi-proximal space if it is

- uniformly/proximally reflexive;
- uniformly/proximally transitive;
- uniformly/proximally fine.

**Definition 2.7.** Let $\mathcal{A}$ be a family of sets, or equivalently $\mathcal{A} \subset \operatorname{Exp}(X)$ for some set $X$. We call

$$\Phi(\mathcal{A}) = \left\{ \bigcap \mathcal{B} : \emptyset \neq \mathcal{B} \subset \mathcal{A}, \text{ and } \mathcal{B} \text{ is finite} \right\}$$

the filtered family of sets generated by $\mathcal{A}$.

Moreover, we say that $\mathcal{A}$ is filtered if $\Phi(\mathcal{A}) = \mathcal{A}$.

**Remark 2.8.** Since $\Phi$ is a refinement for relators on $X$, we write $\mathcal{R}^{\Phi}$ instead of $\Phi(\mathcal{R})$, if $\mathcal{R}$ is a relator on $X$. Note that $\mathcal{R}$ is filtered iff $\mathcal{R}^{\Phi} \subset \mathcal{R}$.

Moreover, note that $\Phi$ is an inversion-compatible refinement for relators on $X$. That is if $\mathcal{R}$ is a relator on $X$, then $\mathcal{R}^{-1\Phi} = \mathcal{R}^{\Phi-1}$.

Finally, if $\mathcal{R}$ is finite, then $\mathcal{R}^{\Phi *} = \{\bigcap \mathcal{R}\}^{*}$.

**Lemma 2.9.** *If $\mathcal{R}$ is a relator on $X$, then $\mathcal{R}^{*\Phi} = \mathcal{R}^{\Phi *}$, $\mathcal{R}^{2\Phi} \subset \mathcal{R}^{\Phi 2 *}$, and $\mathcal{R}^{\#2\#} \subset \mathcal{R}^{\Phi 2 \#}$.*

**Definition 2.10.** If $\square$ is a refinement for relators on $X$, then we say that the $\mathcal{R}$ relator on $X$ is $\square$-filtered if there exists an $\mathcal{S}$ relator on $X$ such that $\mathcal{S}^{\Phi} \subset \mathcal{S}^{\square} = \mathcal{R}^{\square}$.

We use the uniformly filtered and proximally filtered notions instead of $*$-filtered and $\#$-filtered.

**Definition 2.11.** If the generalized uniformity/generalized proximity (generalized quasi-uniformity/generalized quasi-proximity) $\mathcal{R}$ on $X$ is also

- uniformly/proximally filtered,

then we say that $\mathcal{R}$ is a uniformity/proximity (quasi-uniformity/quasi-proximity) on $X$, and $(X, \mathcal{R})$ is a uniform space/proximal space (quasi-uniform space/quasi-proximal space).

# 3. Main example

**Example 3.1.** For all $i \in \mathbb{N}$ let $X_i = \{3i-2, 3i-1, 3i\}$ and $\pi_{i,0}$, $\pi_{i,1}$, ..., $\pi_{i,5}$ are the all bijections of $\{1,2,3\}$ to $X_i$ such that $\pi_{i,0}$ is the only increasing one of them.

Moreover, let

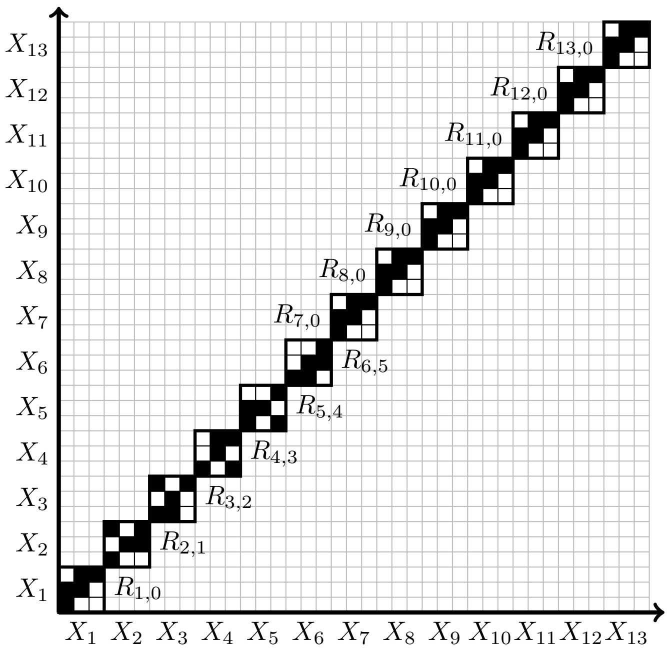
$$R_{i,k} = \{(\pi_{i,k}(1), \pi_{i,k}(2)), (\pi_{i,k}(2), \pi_{i,k}(3))\} \cup \Delta_{X_i}$$

for all $i \in \mathbb{N}$ and $k \in \{0, \dots, 5\}$.

Furthermore, for all $n \in \mathbb{N}$ let $S_n = \bigcup_{i \in \mathbb{N}} R_{i,\nu_i}$, where $\nu_i$ is the $i$th digits of $(n-1)$ in a positional base 6 numeral system, that is $n - 1 = \sum_{i \in \mathbb{N}} \nu_i \cdot 6^{i-1}$.

The following figure shows the main part of the graph of $S_{44791}$, where $44790 = \sum_{i=0}^{5} i \cdot 6^i$ and

$\pi_{2,1}(1) = 4$, $\pi_{2,1}(2) = 6$, $\pi_{3,2}(1) = 8$, $\pi_{3,2}(2) = 7$, $\pi_{4,3}(1) = 11$,
$\pi_{4,3}(2) = 12$, $\pi_{5,4}(1) = 15$, $\pi_{5,4}(2) = 13$, $\pi_{6,5}(1) = 18$, $\pi_{6,5}(2) = 17$.




Finally, let $X = \mathbb{N} = \bigcup_{i \in \mathbb{N}} X_i$ and $\mathcal{R} = \{S_n : n \in \mathbb{N}\}$ is an elementwise reflexive relator on $X$. Then $\mathcal{R}^{\#} = \{\Delta_X\}^*$ is a uniformly transitive relator on $X$, but neither $\mathcal{R}$ nor $\mathcal{R}^{\Phi}$ is proximally transitive. Namely,

$$S_1 = \Delta_X \cup \bigcup_{i \in X \setminus 3X} \{(i, i+1)\} \in \mathcal{R} \subset \mathcal{R}^{\Phi},$$

but we show that if $A = \{3i - 2 : i \in X\}$, then $Q^2[A] \not\subset S_1[A] = A \cup (A+1)$ for all $Q \in \mathcal{R}^{\Phi}$, that is $S_1 \notin \mathcal{R}^{\Phi 2\#}$ and hence $S_1 \notin \mathcal{R}^{2\#}$.

To prove this let $\emptyset \neq \mathcal{S} \subset \mathcal{R}$ finite, such that $Q = \bigcap \mathcal{S}$. Then there exists a greatest $m \in X$ such that $S_m \in \mathcal{S}$. If $i$ is large enough then $\nu_i = 0$ for all elements of $\mathcal{S}$, therefore $R_{i,0} \subset \bigcap \mathcal{S} = Q$, and then $3i \in R_{i,0}^2(3i-2) \subset Q^2(3i-2) \subset Q^2[A]$.

We can change the above example to be symmetric.

**Example 3.2.** Namely, for all $i \in \mathbb{N}$ let $X_i = \{5i-4, 5i-3, 5i-2, 5i-1, 5i\}$ and $\pi_{i,0}$, $\pi_{i,1}$, $\ldots$, $\pi_{i,119}$ are the all bijections of $\{1,2,3,4,5\}$ to $X_i$ such that $\pi_{i,0}$ is the only increasing one of them. Moreover, let

$$R_{i,k} = \{(\pi_{i,k}(1), \pi_{i,k}(2)), (\pi_{i,k}(2), \pi_{i,k}(1)), (\pi_{i,k}(2), \pi_{i,k}(3)), (\pi_{i,k}(3), \pi_{i,k}(2))\} \cup \Delta_{X_i}$$

for all $i \in \mathbb{N}$ and $k \in \{0, \ldots, 119\}$. Furthermore for all $n \in \mathbb{N}$ let $S_n = \bigcup_{i \in \mathbb{N}} R_{i,\nu_i}$, where $\nu_i$ is the $i$th digits of $(n-1)$ in a positional base 120 numeral system, that is $n - 1 = \sum_{i \in \mathbb{N}} \nu_i \cdot 120^{i-1}$. Now $\mathcal{R} = \{S_n : n \in \mathbb{N}\}$ is an elementwise reflexive, elementwise symmetric relator on $X = \mathbb{N} = \bigcup_{i \in \mathbb{N}} X_i$ with the same properties.

## 4. A finite example

**Lemma 4.1.** *Let $\mathcal{R}$ be a finite relator on $X$. Now, $\mathcal{R}^\Phi$ is proximally transitive iff the relation $\bigcap \mathcal{R}$ is transitive.*

***Proof.*** If $\mathcal{R}$ is finite, then Remark 2.4 and 2.8 yield that

$$\mathcal{R}^{\Phi 2*} = \mathcal{R}^{\Phi * 2*} = \left\{\bigcap \mathcal{R}\right\}^{*2*} = \left\{\bigcap \mathcal{R}\right\}^{2*}$$

therefore

$$\mathcal{R}^{\Phi 2\#} = \mathcal{R}^{\Phi 2*\#} = \left\{\bigcap \mathcal{R}\right\}^{2*\#} = \left\{\bigcap \mathcal{R}\right\}^{2\#} = \left\{\bigcap \mathcal{R}\right\}^{2*} = \mathcal{R}^{\Phi 2*}.$$

By using the self-increasingness of $*$ we have that

$$\left(\bigcap \mathcal{R}\right)^2 \subset \bigcap \mathcal{R} \iff \left\{\bigcap \mathcal{R}\right\} \subset \left\{\bigcap \mathcal{R}\right\}^{2*} \iff \left\{\bigcap \mathcal{R}\right\}^{*} \subset \left\{\bigcap \mathcal{R}\right\}^{2*} \iff$$
$$\iff \mathcal{R}^{\Phi *} \subset \mathcal{R}^{\Phi 2*} \iff \mathcal{R}^{\Phi} \subset \mathcal{R}^{\Phi 2*} \iff \mathcal{R}^{\Phi} \subset \mathcal{R}^{\Phi 2\#}. \qquad \square$$

**Theorem 4.2.** *If $\mathcal{R}$ is a finite relator on $X$, then uniform transitivity of $\mathcal{R}^\#$ implies proximal transitivity of $\mathcal{R}^\Phi$.*

***Proof.*** Assume to the contrary that $\mathcal{R}^\Phi$ is not proximally transitive that is, by using the above Lemma, there exists an $(x,z) \in \left(\bigcap \mathcal{R}\right)^2 \setminus \bigcap \mathcal{R}$. It means that there exist $x, y, z \in X$ such that

(a) $y \in (\bigcap \mathcal{R})(x)$;

(b) $z \in (\bigcap \mathcal{R})(y)$;

(c) $z \notin (\bigcap \mathcal{R})(x)$.

For all $S \in \mathcal{R}^{\#}$ there exists an $R \in \mathcal{R}$ such that $R(x) \subset S(x)$. For such an $R \in \mathcal{R}$, by using (a), it is easy to see, that $y \in (\bigcap \mathcal{R})(x) \subset R(x)$.

In a similar way (b) implies that $z \in S(y)$ for all $S \in \mathcal{R}^{\#}$ and (c) implies that $z \notin R(x)$ for some $R \in \mathcal{R}$.

In summary we have that

$$\exists R \in \mathcal{R} : \forall S \in \mathcal{R}^{\#} : S^2 \not\subset R$$

that is $\mathcal{R} \not\subset \mathcal{R}^{\#2*}$. This is a contradiction, because uniform transitivity of $\mathcal{R}^{\#}$ means that $\mathcal{R} \subset \mathcal{R}^{\#} \subset \mathcal{R}^{\#2*}$. □

The following examples show that uniform transitivity of $\mathcal{R}^{\#}$ does not imply proximal transitivity of $\mathcal{R}$ even if the space $X$ is finite.

The appendix shows the graphs of all 24 $R_\pi$ elements of the $\mathcal{R}$ relator in the following example.

Moreover, we can see the graphs of $R_\pi^2$ and $S_n$ elements of $\mathcal{S}$, which is the smallest relator on $X$ such that $\mathcal{S}^* = \mathcal{R}^{\#}$.

Finally, in appendix we can also see $S_k^2 \in \mathcal{S}^2$ examples for all $S_n \in \mathcal{S}$ such that $S_k^2 \subset S_n$.

**Example 4.3.** Let $X = \{1, 2, 3, 4\}$ and

$$R_\pi = \{(\pi(1), \pi(1)), (\pi(1), \pi(2)), (\pi(2), \pi(2)), (\pi(2), \pi(3))\}$$

for all $\pi$ permutation of $X$. Moreover, let $\mathcal{R} = \{R_\pi : \pi \text{ is a permutation of } X\}$.

At first, we investigate $\mathcal{R}^{\#}$. For this end, let $Q \in \mathcal{R}^{\#}$ be arbitrary. By definition, there exists a $\pi$ permutation of $X$, such that $\pi[\{1, 2, 3\}] = R_\pi[X] \subset Q[X]$, that is $Q[X]$ has at least 3 elements. Moreover, if $A \subset X$ has 3 elements, then $\pi(1) \in A$ or $\pi(2) \in A$ when $\pi$ is a permutation of $X$ such that $R_\pi[A] \subset Q[A]$, that is $Q[A]$ has at least 2 elements.

In summary, a $Q$ relation on $X$ is an element of $\mathcal{R}^{\#}$ iff it satisfies at least one of the following conditions.

- There exists $\alpha$ and $\beta$ permutations of $X$ such that $\beta(i) \in Q(\alpha(i))$ for all $i \in \{1, 2, 3\}$.
- There exists $\alpha$ and $\beta$ permutations of $X$ such that $\{\beta(i)\} \subsetneq Q(\alpha(i))$ for all $i \in \{1, 2\}$ and $\beta(3) \in Q(\alpha(1))$.

Now, we show that $\mathcal{R}^{\#}$ is uniformly transitive. For this end, let $Q \in \mathcal{R}^{\#}$ be arbitrary. This is possible in the following ways.

<u>$Q \subset \Delta_X$</u>:

(1) There exists an $\alpha$ permutation of $X$ such that $(\alpha(1), \alpha(1))$, $(\alpha(2), \alpha(2)) \in Q$. In this case

$$P = \{(\alpha(1), \alpha(1)), (\alpha(2), \alpha(2)), (\alpha(3), \alpha(4))\} \in \mathcal{R}^{\#} \text{ and } P^2 \subset Q.$$

$\underline{Q \subset X^2 \setminus \Delta_X}$:

(2) There exists an $\alpha$ permutation of $X$ such that $(\alpha(1), \alpha(2))$, $(\alpha(1), \alpha(3)) \in Q$. In this case

$$P = \{(\alpha(1), \alpha(2)), (\alpha(1), \alpha(4)), (\alpha(4), \alpha(2)), (\alpha(4), \alpha(3))\} \in \mathcal{R}^{\#} \text{ and } P^2 \subset Q.$$

Or

(3) there exists an $\alpha$ permutation of $X$ such that $(\alpha(1), \alpha(2))$, $(\alpha(3), \alpha(4)) \in Q$. In this case

$$P = \{(\alpha(1), \alpha(3)), (\alpha(2), \alpha(4)), (\alpha(3), \alpha(2))\} \in \mathcal{R}^{\#} \text{ and } P^2 \subset Q.$$

Or

(4) there exists an $\alpha$ permutation of $X$ such that $(\alpha(1), \alpha(2))$, $(\alpha(2), \alpha(3))$, $(\alpha(3), \alpha(1)) \in Q$. In this case

$$P = \{(\alpha(1), \alpha(3)), (\alpha(2), \alpha(1)), (\alpha(3), \alpha(2))\} \in \mathcal{R}^{\#} \text{ and } P^2 \subset Q.$$

$\underline{Q \not\subset \Delta_X \text{ and } Q \not\subset X^2 \setminus \Delta_X}$:

(5) There exists an $\alpha$ permutation of $X$ such that $(\alpha(1), \alpha(1))$, $(\alpha(1), \alpha(2)) \in Q$. In this case

$$P = \{(\alpha(1), \alpha(1)), (\alpha(1), \alpha(3)), (\alpha(2), \alpha(4)), (\alpha(3), \alpha(4))\} \in \mathcal{R}^{\#} \text{ and } P^2 \subset Q.$$

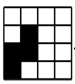
Or

(6) there exists an $\alpha$ permutation of $X$ such that $(\alpha(1), \alpha(1))$, $(\alpha(2), \alpha(3)) \in Q$. In this case

$$P = \{(\alpha(1), \alpha(1)), (\alpha(2), \alpha(4)), (\alpha(4), \alpha(3))\} \in \mathcal{R}^{\#} \text{ and } P^2 \subset Q.$$

Finally, we show that $\mathcal{R}$ is not proximally transitive. For this, note that for any $\pi$ permutation of $X$

$$R_\pi^2[\{1,3,4\}] \not\subset \{1,2\} = R_{\Delta_X}[\{1,3,4\}]$$

Note that $R_{\Delta_X} =$ .

Unfortunately, the above relator is neither reflexive nor symmetric. We can make a reflexive one.

**Example 4.4.** Let $X = \{1,2,3,4\}$, $Y = \{1,2,3,4,5,6,7,8\}$ and

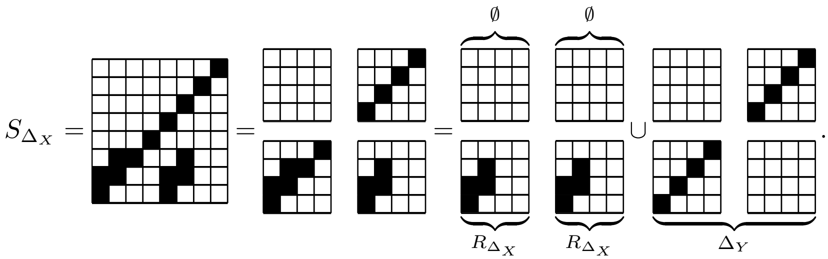
$$R_\pi = \{(\pi(1), \pi(1)), (\pi(1), \pi(2)), (\pi(2), \pi(2)), (\pi(2), \pi(3))\}$$

and

$$S_\pi = R_\pi \cup \{(\pi(1)+4, \pi(1)), (\pi(1)+4, \pi(2)), (\pi(2)+4, \pi(2)), (\pi(2)+4, \pi(3))\} \cup \Delta_Y$$

for all $\pi$ permutation of $X$.

Moreover, let $\mathcal{S} = \{S_\pi : \pi \text{ is a permutation of } X\}$ be an elementwise reflexive relator on $Y$. Now $\mathcal{S}^\#$ is uniformly transitive, but $\mathcal{S}$ is not proximally transitive. Note that




We do not know whether if a symmetric finite $\mathcal{R}$ relator is not proximally transitive, then can $\mathcal{R}^\#$ be uniformly transitive.

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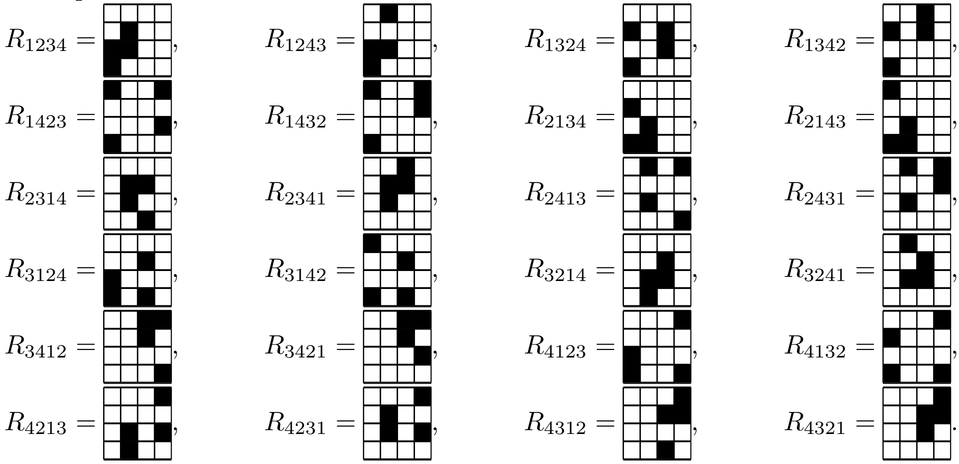
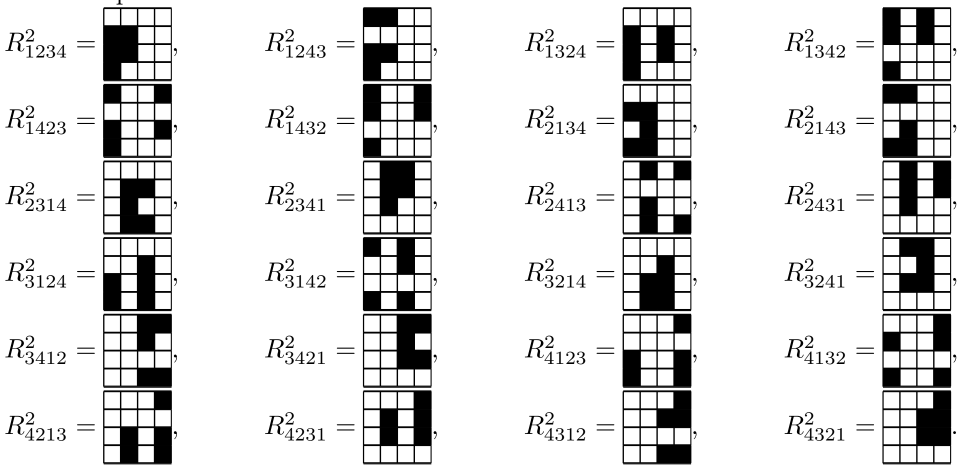
# Appendix

List of elements of $\mathcal{R}$ of Example 4.3, with using the notation $\pi(1)\pi(2)\pi(3)\pi(4)$ for the $\pi$ permutation of $X$.

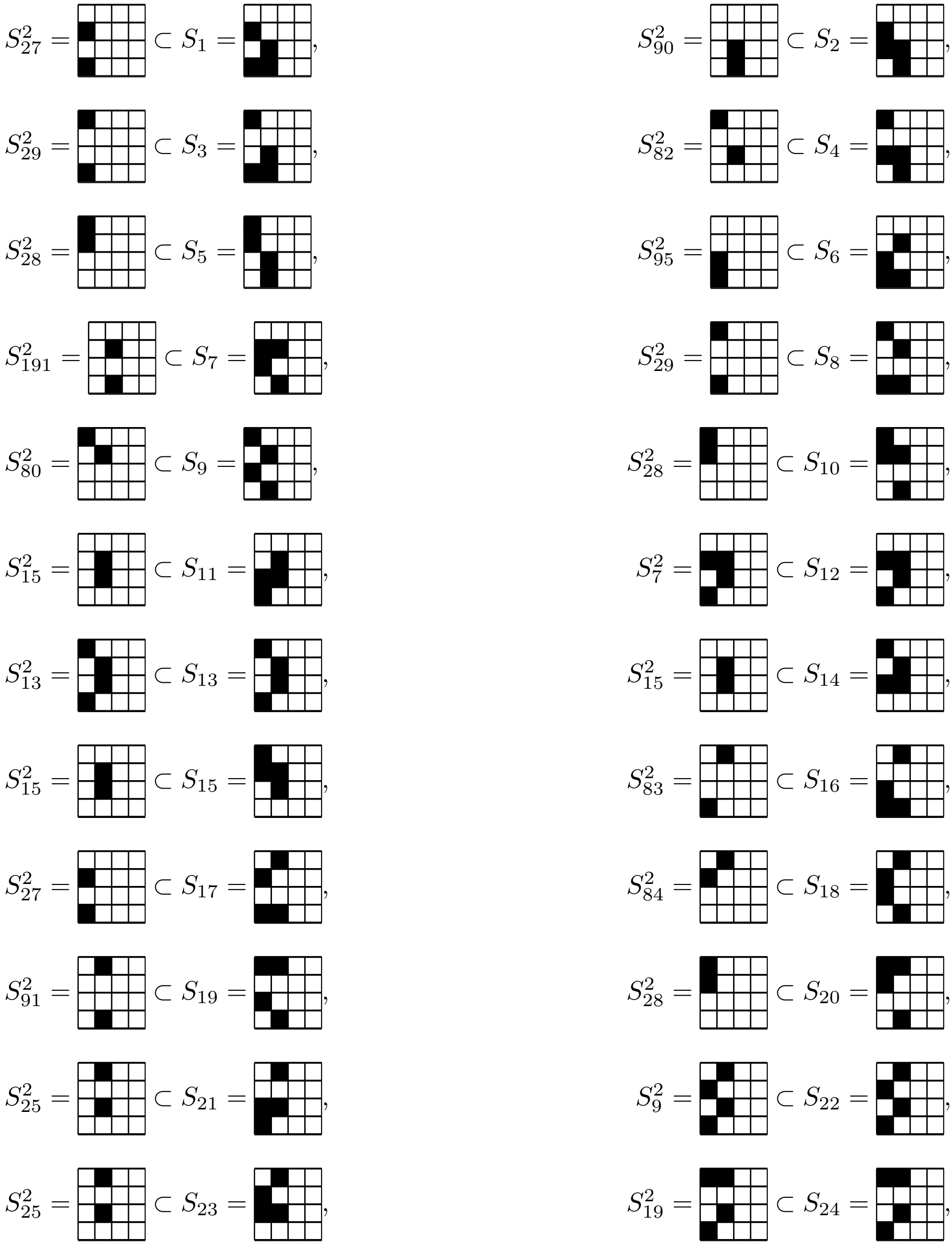



List of elements of $\mathcal{R}^2$ of Example 4.3, with using the notation $\pi(1)\pi(2)\pi(3)\pi(4)$ for the $\pi$ permutation of $X$.



In Example 4.3, $\mathcal{S}$ is the smallest relator on $X$, such that $\mathcal{S}^* = \mathcal{R}^\#$

We draw the graph of elements of $\mathcal{S}$ with marked the required member of $\mathcal{S}$, which implies the $\mathcal{S} \subset \mathcal{S}^{2*}$ inclusion.



$$S_{25}^2 = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{25} = \begin{pmatrix} 1&1&0&0\\ 1&0&0&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix},$$

$$S_{10}^2 = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{26} = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 1&0&0&0\\ 1&0&0&0 \end{pmatrix},$$

$$S_{10}^2 = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{27} = \begin{pmatrix} 0&1&0&0\\ 1&1&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix},$$

$$S_{10}^2 = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{28} = \begin{pmatrix} 0&1&0&0\\ 1&1&0&0\\ 1&0&0&0\\ 0&0&0&0 \end{pmatrix},$$

$$S_{10}^2 = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{29} = \begin{pmatrix} 1&1&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix},$$

$$S_{10}^2 = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{30} = \begin{pmatrix} 1&1&0&0\\ 0&1&0&0\\ 1&0&0&0\\ 0&0&0&0 \end{pmatrix},$$

$$S_{131}^2 = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&1&0&0\\ 0&0&1&0 \end{pmatrix} \subset S_{31} = \begin{pmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{82}^2 = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{32} = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&1&0&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{37}^2 = \begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 1&0&0&0\\ 0&0&1&0 \end{pmatrix} \subset S_{33} = \begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 1&0&0&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{80}^2 = \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{34} = \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{38}^2 = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0 \end{pmatrix} \subset S_{35} = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 1&0&0&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{38}^2 = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0 \end{pmatrix} \subset S_{36} = \begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&0&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{33}^2 = \begin{pmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&1&0&0 \end{pmatrix} \subset S_{37} = \begin{pmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&1&0&0 \end{pmatrix},$$

$$S_{35}^2 = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&0&0 \end{pmatrix} \subset S_{38} = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&1&0&0 \end{pmatrix},$$

$$S_{147}^2 = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 1&0&0&0 \end{pmatrix} \subset S_{39} = \begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 1&0&0&0 \end{pmatrix},$$

$$S_{35}^2 = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&0&0 \end{pmatrix} \subset S_{40} = \begin{pmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&0 \end{pmatrix},$$

$$S_{83}^2 = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix} \subset S_{41} = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 1&0&0&0 \end{pmatrix},$$

$$S_{84}^2 = \begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{42} = \begin{pmatrix} 0&1&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&0&0&0 \end{pmatrix},$$

$$S_{27}^2 = \begin{pmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix} \subset S_{43} = \begin{pmatrix} 0&0&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 1&0&1&0 \end{pmatrix},$$

$$S_{163}^2 = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&1&0 \end{pmatrix} \subset S_{44} = \begin{pmatrix} 0&0&0&0\\ 1&0&0&0\\ 1&0&1&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{29}^2 = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{pmatrix} \subset S_{45} = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 1&0&1&0 \end{pmatrix},$$

$$S_{35}^2 = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&0&0 \end{pmatrix} \subset S_{46} = \begin{pmatrix} 1&0&0&0\\ 0&0&0&0\\ 1&0&1&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{28}^2 = \begin{pmatrix} 1&0&0&0\\ 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{47} = \begin{pmatrix} 1&0&0&0\\ 1&0&0&0\\ 0&0&1&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{163}^2 = \begin{pmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&0&1&0 \end{pmatrix} \subset S_{48} = \begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&1&1&0 \end{pmatrix},$$

$$S_{15}^2 = \begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{49} = \begin{pmatrix} 0&0&0&0\\ 0&1&0&0\\ 0&1&1&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{38}^2 = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&1&0 \end{pmatrix} \subset S_{50} = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&0&1&0\\ 0&1&1&0 \end{pmatrix},$$

$$S_{25}^2 = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&1&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{51} = \begin{pmatrix} 0&1&0&0\\ 0&0&0&0\\ 0&1&1&0\\ 0&0&1&0 \end{pmatrix},$$

$$S_{10}^2 = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{pmatrix} \subset S_{52} = \begin{pmatrix} 0&1&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&1&0 \end{pmatrix},$$

$$S^2_{128} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{53} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{31} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{55} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{36} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{57} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{36} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{59} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline \blacksquare & & \blacksquare & \\\hline\end{array},$$

$$S^2_{29} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{61} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & \blacksquare & \\\hline\end{array},$$

$$S^2_{28} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{63} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline \blacksquare & & \blacksquare & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{15} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{65} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & \blacksquare & \blacksquare & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{25} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{67} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{36} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{69} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & \blacksquare & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{29} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{71} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{28} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{73} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline \blacksquare & & \blacksquare & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array},$$

$$S^2_{72} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array} \subset S_{75} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & \blacksquare & \blacksquare & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{25} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{77} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & \blacksquare & \blacksquare & \\\hline & & & \\\hline\end{array},$$

$$S^2_{58} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{54} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{40} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{56} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array},$$

$$S^2_{54} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{58} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array},$$

$$S^2_{66} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array} \subset S_{60} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{58} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{62} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{40} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{64} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & \blacksquare & \blacksquare & \\\hline\end{array},$$

$$S^2_{38} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array} \subset S_{66} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & \blacksquare & \\\hline\end{array},$$

$$S^2_{10} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{68} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & \blacksquare & \blacksquare & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{27} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{70} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & \blacksquare & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{35} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array} \subset S_{72} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & \blacksquare & \\\hline & & & \\\hline\end{array},$$

$$S^2_{40} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{74} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & \blacksquare & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{54} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{76} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{10} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{78} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & \blacksquare & \blacksquare & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array},$$

$S_{34}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{79} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline & \blacksquare & & \\\hline\end{array}$, $\quad S_{34}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{80} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array}$,

$S_{32}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{81} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline \blacksquare & & & \\\hline\end{array}$, $\quad S_{32}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{82} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array}$,

$S_{41}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{83} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array}$, $\quad S_{42}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{84} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array}$,

$S_{41}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{85} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline \blacksquare & & \blacksquare & \\\hline\end{array}$, $\quad S_{27}^2 = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{86} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & \blacksquare & \\\hline\end{array}$,

$S_{42}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{87} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline\end{array}$, $\quad S_{42}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{88} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline\end{array}$,

$S_{28}^2 = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{89} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array}$, $\quad S_{32}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{90} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline & \blacksquare & \blacksquare & \\\hline\end{array}$,

$S_{34}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{91} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline & \blacksquare & \blacksquare & \\\hline\end{array}$, $\quad S_{15}^2 = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{92} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline\end{array}$,

$S_{25}^2 = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{93} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline\end{array}$, $\quad S_{10}^2 = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{94} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array}$,

$S_{41}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{95} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & \blacksquare & \\\hline \blacksquare & & & \\\hline\end{array}$, $\quad S_{27}^2 = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{96} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline\end{array}$,

$S_{42}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{97} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline \blacksquare & & \blacksquare & \\\hline & & & \\\hline\end{array}$, $\quad S_{29}^2 = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{98} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & \blacksquare & \\\hline & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline\end{array}$,

$S_{28}^2 = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{99} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array}$, $\quad S_{32}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{100} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & \blacksquare & \\\hline & \blacksquare & & \\\hline\end{array}$,

$S_{34}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{101} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline\end{array}$, $\quad S_{15}^2 = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{102} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & \blacksquare & \blacksquare & \\\hline & & & \\\hline\end{array}$,

$S_{34}^2 = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{103} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & \blacksquare & \\\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline\end{array}$, $\quad S_{10}^2 = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{104} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & \blacksquare & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array}$,

$S_{36}^2 = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{105} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline\end{array}$, $\quad S_{27}^2 = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{106} = \begin{array}{|c|c|c|c|}\hline & & \blacksquare & \\\hline \blacksquare & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array}$,

$$S^2_{42} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{107} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \blacksquare & \square & \blacksquare & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{array}, \qquad S^2_{29} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{array} \subset S_{108} = \begin{array}{cccc} \blacksquare & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{array},$$

$$S^2_{42} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{109} = \begin{array}{cccc} \blacksquare & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{array}, \qquad S^2_{32} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{110} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \end{array},$$

$$S^2_{34} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{array} \subset S_{111} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \blacksquare & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{array}, \qquad S^2_{15} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{112} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \blacksquare & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array},$$

$$S^2_{34} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{array} \subset S_{113} = \begin{array}{cccc} \square & \blacksquare & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{array}, \qquad S^2_{25} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{114} = \begin{array}{cccc} \square & \blacksquare & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array},$$

$$S^2_{169} = \begin{array}{cccc} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{115} = \begin{array}{cccc} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{82} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{116} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{133} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array} \subset S_{117} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{80} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{118} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{134} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array} \subset S_{119} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{84} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{120} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{149} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{array} \subset S_{121} = \begin{array}{cccc} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{35} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \end{array} \subset S_{122} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{133} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array} \subset S_{123} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{149} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{array} \subset S_{124} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{144} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{125} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{58} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{126} = \begin{array}{cccc} \blacksquare & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{40} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{127} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{54} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{128} = \begin{array}{cccc} \square & \blacksquare & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{42} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{129} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{173} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array} \subset S_{130} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{32} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{array} \subset S_{131} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \end{array}, \qquad S^2_{133} = \begin{array}{cccc} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array} \subset S_{132} = \begin{array}{cccc} \square & \square & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{array},$$

$$S^2_{117} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix} \subset S_{133} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{bmatrix},$$

$$S^2_{170} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{bmatrix} \subset S_{135} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \end{bmatrix},$$

$$S^2_{83} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{bmatrix} \subset S_{137} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \end{bmatrix},$$

$$S^2_{117} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix} \subset S_{139} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{bmatrix},$$

$$S^2_{129} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{bmatrix} \subset S_{141} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{bmatrix},$$

$$S^2_{36} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{bmatrix} \subset S_{143} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \end{bmatrix},$$

$$S^2_{125} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix} \subset S_{145} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{bmatrix},$$

$$S^2_{41} = \begin{bmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{bmatrix} \subset S_{147} = \begin{bmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \end{bmatrix},$$

$$S^2_{34} = \begin{bmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{bmatrix} \subset S_{149} = \begin{bmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{bmatrix},$$

$$S^2_{27} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{bmatrix} \subset S_{151} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \blacksquare \end{bmatrix},$$

$$S^2_{29} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{bmatrix} \subset S_{153} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \blacksquare \end{bmatrix},$$

$$S^2_{28} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{155} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \end{bmatrix},$$

$$S^2_{15} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{157} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \blacksquare \\ \square & \square & \square & \blacksquare \end{bmatrix},$$

$$S^2_{25} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{159} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \blacksquare \\ \square & \square & \square & \blacksquare \end{bmatrix},$$

$$S^2_{119} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{bmatrix} \subset S_{134} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{bmatrix},$$

$$S^2_{80} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{136} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix},$$

$$S^2_{84} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{138} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix},$$

$$S^2_{129} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{bmatrix} \subset S_{140} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{bmatrix},$$

$$S^2_{38} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \end{bmatrix} \subset S_{142} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{bmatrix},$$

$$S^2_{58} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{144} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix},$$

$$S^2_{54} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{146} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix},$$

$$S^2_{117} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix} \subset S_{148} = \begin{bmatrix} \square & \square & \blacksquare & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix},$$

$$S^2_{179} = \begin{bmatrix} \square & \square & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix} \subset S_{150} = \begin{bmatrix} \square & \square & \blacksquare & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix},$$

$$S^2_{117} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{bmatrix} \subset S_{152} = \begin{bmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \end{bmatrix},$$

$$S^2_{97} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{154} = \begin{bmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \end{bmatrix},$$

$$S^2_{133} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{bmatrix} \subset S_{156} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \blacksquare \end{bmatrix},$$

$$S^2_{91} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{bmatrix} \subset S_{158} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \blacksquare \end{bmatrix},$$

$$S^2_{10} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{bmatrix} \subset S_{160} = \begin{bmatrix} \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \end{bmatrix},$$

$$S_{66}^2 = \begin{bmatrix}0&0&0&0\\0&0&1&0\\0&0&0&0\\0&0&1&0\end{bmatrix} \subset S_{161} = \begin{bmatrix}0&0&0&0\\0&0&1&0\\0&0&0&1\\0&0&1&1\end{bmatrix},$$

$$S_{50}^2 = \begin{bmatrix}0&0&1&0\\0&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix} \subset S_{163} = \begin{bmatrix}0&0&1&0\\0&0&0&0\\0&0&0&1\\0&0&1&1\end{bmatrix},$$

$$S_{109}^2 = \begin{bmatrix}0&0&1&0\\0&0&1&0\\0&0&0&0\\0&0&0&0\end{bmatrix} \subset S_{165} = \begin{bmatrix}0&0&1&0\\0&0&1&0\\0&0&0&1\\0&0&0&1\end{bmatrix},$$

$$S_{120}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{bmatrix} \subset S_{167} = \begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{bmatrix},$$

$$S_{82}^2 = \begin{bmatrix}1&0&0&0\\0&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix} \subset S_{169} = \begin{bmatrix}1&0&0&0\\0&0&0&1\\0&1&0&0\\0&0&0&0\end{bmatrix},$$

$$S_{121}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\0&0&0&0\end{bmatrix} \subset S_{171} = \begin{bmatrix}0&1&0&0\\0&0&0&1\\1&0&0&0\\0&0&0&0\end{bmatrix},$$

$$S_{130}^2 = \begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&0&0\\0&0&1&0\end{bmatrix} \subset S_{173} = \begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&0&0\\0&0&1&0\end{bmatrix},$$

$$S_{38}^2 = \begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix} \subset S_{175} = \begin{bmatrix}0&1&0&0\\0&0&0&1\\0&0&0&0\\0&0&1&0\end{bmatrix},$$

$$S_{35}^2 = \begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix} \subset S_{177} = \begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&0&0&0\end{bmatrix},$$

$$S_{150}^2 = \begin{bmatrix}0&1&0&0\\0&0&0&1\\0&0&1&0\\0&0&0&0\end{bmatrix} \subset S_{179} = \begin{bmatrix}0&1&0&0\\0&0&0&1\\0&0&1&0\\0&0&0&0\end{bmatrix},$$

$$S_{42}^2 = \begin{bmatrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{bmatrix} \subset S_{181} = \begin{bmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&0&0&0\end{bmatrix},$$

$$S_{32}^2 = \begin{bmatrix}0&0&1&0\\0&0&0&0\\0&1&0&0\\0&0&0&0\end{bmatrix} \subset S_{183} = \begin{bmatrix}0&0&1&0\\0&0&0&1\\0&1&0&0\\0&0&0&0\end{bmatrix},$$

$$S_{121}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\0&0&0&0\end{bmatrix} \subset S_{185} = \begin{bmatrix}0&0&0&0\\1&0&0&1\\1&0&0&0\\0&0&0&1\end{bmatrix},$$

$$S_{72}^2 = \begin{bmatrix}0&0&0&0\\0&0&1&0\\0&0&1&0\\0&0&0&0\end{bmatrix} \subset S_{162} = \begin{bmatrix}0&0&0&0\\0&0&1&0\\0&0&1&1\\0&0&0&1\end{bmatrix},$$

$$S_{46}^2 = \begin{bmatrix}0&0&1&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix} \subset S_{164} = \begin{bmatrix}0&0&1&0\\0&0&0&0\\0&0&1&1\\0&0&0&1\end{bmatrix},$$

$$S_{120}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{bmatrix} \subset S_{166} = \begin{bmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{bmatrix},$$

$$S_{79}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&0\\0&1&0&0\\1&0&0&0\end{bmatrix} \subset S_{168} = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&1&0&0\\1&0&0&0\end{bmatrix},$$

$$S_{83}^2 = \begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix} \subset S_{170} = \begin{bmatrix}0&1&0&0\\0&0&0&1\\0&0&0&0\\1&0&0&0\end{bmatrix},$$

$$S_{121}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\0&0&0&0\end{bmatrix} \subset S_{172} = \begin{bmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\0&0&1&0\end{bmatrix},$$

$$S_{115}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&1&0&0\\0&0&0&0\end{bmatrix} \subset S_{174} = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&1&0&0\\0&0&1&0\end{bmatrix},$$

$$S_{135}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\1&0&0&0\end{bmatrix} \subset S_{176} = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&1&0\\1&0&0&0\end{bmatrix},$$

$$S_{120}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{bmatrix} \subset S_{178} = \begin{bmatrix}0&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{bmatrix},$$

$$S_{41}^2 = \begin{bmatrix}0&0&1&0\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix} \subset S_{180} = \begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\1&0&0&0\end{bmatrix},$$

$$S_{34}^2 = \begin{bmatrix}0&0&1&0\\0&0&0&0\\0&0&0&0\\0&1&0&0\end{bmatrix} \subset S_{182} = \begin{bmatrix}0&0&1&0\\0&0&0&1\\0&0&0&0\\0&1&0&0\end{bmatrix},$$

$$S_{95}^2 = \begin{bmatrix}0&0&0&0\\0&0&0&0\\1&0&0&0\\1&0&0&0\end{bmatrix} \subset S_{184} = \begin{bmatrix}0&0&0&0\\0&0&0&1\\1&0&0&0\\1&0&0&1\end{bmatrix},$$

$$S_{29}^2 = \begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{bmatrix} \subset S_{186} = \begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&0&0\\1&0&0&1\end{bmatrix},$$

$$S^2_{97} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{187} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{28} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{188} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \blacksquare \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{90} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \end{smallmatrix} \subset S_{189} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{15} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{190} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \blacksquare \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{91} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{smallmatrix} \subset S_{191} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{25} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{192} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{10} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{193} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \blacksquare \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{149} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix} \subset S_{194} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \blacksquare \end{smallmatrix},$$

$$S^2_{72} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{195} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \blacksquare \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{50} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \end{smallmatrix} \subset S_{196} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \blacksquare \end{smallmatrix},$$

$$S^2_{46} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{197} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{109} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{198} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \blacksquare \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \end{smallmatrix},$$

$$S^2_{95} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{smallmatrix} \subset S_{199} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \end{smallmatrix},$$

$$S^2_{27} = \begin{smallmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{smallmatrix} \subset S_{200} = \begin{smallmatrix} \square & \square & \square & \square \\ \blacksquare & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \end{smallmatrix},$$

$$S^2_{29} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \end{smallmatrix} \subset S_{201} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \square \end{smallmatrix},$$

$$S^2_{97} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{202} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \blacksquare & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{smallmatrix},$$

$$S^2_{28} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{203} = \begin{smallmatrix} \blacksquare & \square & \square & \square \\ \blacksquare & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{smallmatrix},$$

$$S^2_{90} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \end{smallmatrix} \subset S_{204} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{smallmatrix},$$

$$S^2_{120} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{smallmatrix} \subset S_{205} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \blacksquare & \square & \blacksquare \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{smallmatrix},$$

$$S^2_{91} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{smallmatrix} \subset S_{206} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \square \end{smallmatrix},$$

$$S^2_{25} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{207} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \blacksquare & \square & \blacksquare \\ \square & \square & \square & \square \end{smallmatrix},$$

$$S^2_{10} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{208} = \begin{smallmatrix} \square & \blacksquare & \square & \square \\ \square & \blacksquare & \square & \blacksquare \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{smallmatrix},$$

$$S^2_{129} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{smallmatrix} \subset S_{209} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \blacksquare \\ \square & \square & \blacksquare & \square \end{smallmatrix},$$

$$S^2_{66} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \end{smallmatrix} \subset S_{210} = \begin{smallmatrix} \square & \square & \square & \square \\ \square & \square & \blacksquare & \blacksquare \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{smallmatrix},$$

$$S^2_{50} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \end{smallmatrix} \subset S_{211} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \square \end{smallmatrix},$$

$$S^2_{46} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{212} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \square & \blacksquare \\ \square & \square & \blacksquare & \blacksquare \\ \square & \square & \square & \square \end{smallmatrix},$$

$$S^2_{109} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \square \\ \square & \square & \square & \square \\ \square & \square & \square & \square \end{smallmatrix} \subset S_{213} = \begin{smallmatrix} \square & \square & \blacksquare & \square \\ \square & \square & \blacksquare & \blacksquare \\ \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \end{smallmatrix},$$

$$S^2_{223} = \begin{smallmatrix} \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \\ \square & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{smallmatrix} \subset S_{214} = \begin{smallmatrix} \square & \square & \square & \blacksquare \\ \square & \square & \square & \square \\ \blacksquare & \square & \square & \square \\ \square & \blacksquare & \square & \square \end{smallmatrix},$$

$$S^2_{219} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{215} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{79} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{216} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & \blacksquare & & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{138} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{217} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array},$$

$$S^2_{118} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{218} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{215} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{219} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array},$$

$$S^2_{225} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{220} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{219} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{221} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{131} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline\end{array} \subset S_{222} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{215} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{223} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & & & \\\hline & & \blacksquare & \\\hline\end{array},$$

$$S^2_{118} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{224} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{219} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{225} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array},$$

$$S^2_{220} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array} \subset S_{226} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{215} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{227} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & & \blacksquare & \\\hline & & & \\\hline\end{array},$$

$$S^2_{36} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{228} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & \blacksquare & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array},$$

$$S^2_{144} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{229} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & \blacksquare & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array},$$

$$S^2_{128} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{230} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & \blacksquare & \\\hline & & & \\\hline & \blacksquare & & \\\hline\end{array},$$

$$S^2_{40} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{231} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & \blacksquare & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array},$$

$$S^2_{95} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{232} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline \blacksquare & & & \\\hline \blacksquare & & & \blacksquare \\\hline\end{array},$$

$$S^2_{27} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline\end{array} \subset S_{233} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & & \blacksquare \\\hline\end{array},$$

$$S^2_{202} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{234} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & & \blacksquare \\\hline\end{array},$$

$$S^2_{97} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline\end{array} \subset S_{235} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \blacksquare \\\hline & & & \\\hline \blacksquare & & & \\\hline & & & \blacksquare \\\hline\end{array},$$

$$S^2_{28} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \\\hline\end{array} \subset S_{236} = \begin{array}{|c|c|c|c|}\hline \blacksquare & & & \blacksquare \\\hline \blacksquare & & & \\\hline & & & \\\hline & & & \blacksquare \\\hline\end{array},$$

$$S^2_{90} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & & & \\\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline\end{array} \subset S_{237} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & & & \\\hline & \blacksquare & & \\\hline & \blacksquare & & \blacksquare \\\hline\end{array},$$

$$S^2_{133} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline & & & \blacksquare \\\hline\end{array} \subset S_{238} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & & & \\\hline & \blacksquare & & \blacksquare \\\hline\end{array},$$

$$S^2_{15} = \begin{array}{|c|c|c|c|}\hline & & & \\\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{239} = \begin{array}{|c|c|c|c|}\hline & & & \blacksquare \\\hline & \blacksquare & & \\\hline & \blacksquare & & \\\hline & & & \blacksquare \\\hline\end{array},$$

$$S^2_{25} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \\\hline\end{array} \subset S_{240} = \begin{array}{|c|c|c|c|}\hline & \blacksquare & & \blacksquare \\\hline & & & \\\hline & \blacksquare & & \\\hline & & & \blacksquare \\\hline\end{array},$$

$S^2_{10} = \begin{bmatrix}\square&\blacksquare&\square&\square\\\square&\blacksquare&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{241} = \begin{bmatrix}\square&\blacksquare&\square&\blacksquare\\\square&\blacksquare&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\blacksquare\end{bmatrix},$ $S^2_{149} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\blacksquare\end{bmatrix} \subset S_{242} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\blacksquare&\blacksquare\end{bmatrix},$

$S^2_{66} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\blacksquare&\square\end{bmatrix} \subset S_{243} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\blacksquare&\blacksquare\end{bmatrix},$ $S^2_{72} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{244} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\blacksquare&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\blacksquare\end{bmatrix},$

$S^2_{46} = \begin{bmatrix}\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{245} = \begin{bmatrix}\square&\square&\blacksquare&\blacksquare\\\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\blacksquare\end{bmatrix},$ $S^2_{109} = \begin{bmatrix}\square&\square&\blacksquare&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{246} = \begin{bmatrix}\square&\square&\blacksquare&\blacksquare\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\square&\blacksquare\end{bmatrix},$

$S^2_{95} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix} \subset S_{247} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\square&\square\\\blacksquare&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\end{bmatrix},$ $S^2_{27} = \begin{bmatrix}\square&\square&\square&\square\\\blacksquare&\square&\square&\square\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix} \subset S_{248} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\\\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\end{bmatrix},$

$S^2_{117} = \begin{bmatrix}\square&\square&\square&\square\\\blacksquare&\square&\square&\square\\\square&\square&\square&\blacksquare\\\square&\square&\square&\square\end{bmatrix} \subset S_{249} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\\\blacksquare&\square&\square&\blacksquare\\\square&\square&\square&\square\end{bmatrix},$ $S^2_{29} = \begin{bmatrix}\blacksquare&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix} \subset S_{250} = \begin{bmatrix}\blacksquare&\square&\square&\blacksquare\\\square&\square&\square&\square\\\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\end{bmatrix},$

$S^2_{28} = \begin{bmatrix}\blacksquare&\square&\square&\square\\\blacksquare&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{251} = \begin{bmatrix}\blacksquare&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\\\square&\square&\square&\blacksquare\\\square&\square&\square&\square\end{bmatrix},$ $S^2_{90} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\blacksquare&\square&\square\\\square&\blacksquare&\square&\square\end{bmatrix} \subset S_{252} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\square&\square\\\square&\blacksquare&\square&\blacksquare\\\square&\blacksquare&\square&\square\end{bmatrix},$

$S^2_{191} = \begin{bmatrix}\square&\square&\square&\square\\\square&\blacksquare&\square&\square\\\square&\square&\square&\square\\\square&\blacksquare&\square&\square\end{bmatrix} \subset S_{253} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\blacksquare&\square&\square\\\square&\square&\square&\blacksquare\\\square&\blacksquare&\square&\square\end{bmatrix},$ $S^2_{15} = \begin{bmatrix}\square&\square&\square&\square\\\square&\blacksquare&\square&\square\\\square&\blacksquare&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{254} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\blacksquare&\square&\square\\\square&\blacksquare&\square&\blacksquare\\\square&\square&\square&\square\end{bmatrix},$

$S^2_{91} = \begin{bmatrix}\square&\blacksquare&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\blacksquare&\square&\square\end{bmatrix} \subset S_{255} = \begin{bmatrix}\square&\blacksquare&\square&\blacksquare\\\square&\square&\square&\square\\\square&\square&\square&\blacksquare\\\square&\blacksquare&\square&\square\end{bmatrix},$ $S^2_{10} = \begin{bmatrix}\square&\blacksquare&\square&\square\\\square&\blacksquare&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{256} = \begin{bmatrix}\square&\blacksquare&\square&\blacksquare\\\square&\blacksquare&\square&\square\\\square&\square&\square&\blacksquare\\\square&\square&\square&\square\end{bmatrix},$

$S^2_{129} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\blacksquare\\\square&\square&\blacksquare&\square\end{bmatrix} \subset S_{257} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\square&\square\\\square&\square&\blacksquare&\blacksquare\\\square&\square&\blacksquare&\square\end{bmatrix},$ $S^2_{66} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\blacksquare&\square\end{bmatrix} \subset S_{258} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\blacksquare&\square\\\square&\square&\square&\blacksquare\\\square&\square&\blacksquare&\square\end{bmatrix},$

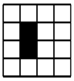
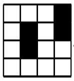
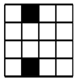
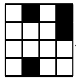
$S^2_{72} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\blacksquare&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{259} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\blacksquare&\square\\\square&\square&\blacksquare&\blacksquare\\\square&\square&\square&\square\end{bmatrix},$ $S^2_{50} = \begin{bmatrix}\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\blacksquare&\square\end{bmatrix} \subset S_{260} = \begin{bmatrix}\square&\square&\blacksquare&\blacksquare\\\square&\square&\square&\square\\\square&\square&\square&\blacksquare\\\square&\square&\blacksquare&\square\end{bmatrix},$

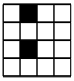
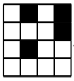
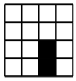
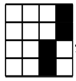
$S^2_{109} = \begin{bmatrix}\square&\square&\blacksquare&\square\\\square&\square&\blacksquare&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{261} = \begin{bmatrix}\square&\square&\blacksquare&\blacksquare\\\square&\square&\blacksquare&\square\\\square&\square&\square&\blacksquare\\\square&\square&\square&\square\end{bmatrix},$ $S^2_{95} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix} \subset S_{262} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix},$

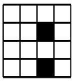
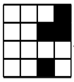
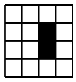
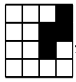
$S^2_{27} = \begin{bmatrix}\square&\square&\square&\square\\\blacksquare&\square&\square&\square\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix} \subset S_{263} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\blacksquare\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix},$ $S^2_{121} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{264} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix},$

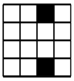
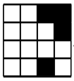
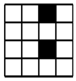
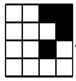
$S^2_{29} = \begin{bmatrix}\blacksquare&\square&\square&\square\\\square&\square&\square&\square\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix} \subset S_{265} = \begin{bmatrix}\blacksquare&\square&\square&\blacksquare\\\square&\square&\square&\blacksquare\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\end{bmatrix},$ $S^2_{97} = \begin{bmatrix}\blacksquare&\square&\square&\square\\\square&\square&\square&\square\\\blacksquare&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix} \subset S_{266} = \begin{bmatrix}\blacksquare&\square&\square&\blacksquare\\\square&\square&\square&\blacksquare\\\blacksquare&\square&\square&\square\\\square&\square&\square&\square\end{bmatrix},$

$S^2_{90} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\square\\\square&\blacksquare&\square&\square\\\square&\blacksquare&\square&\square\end{bmatrix} \subset S_{267} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\square&\square&\blacksquare\\\square&\blacksquare&\square&\square\\\square&\blacksquare&\square&\square\end{bmatrix},$ $S^2_{120} = \begin{bmatrix}\square&\square&\square&\square\\\square&\square&\square&\blacksquare\\\square&\square&\square&\square\\\square&\blacksquare&\square&\square\end{bmatrix} \subset S_{268} = \begin{bmatrix}\square&\square&\square&\blacksquare\\\square&\blacksquare&\square&\blacksquare\\\square&\square&\square&\square\\\square&\blacksquare&\square&\square\end{bmatrix},$

$S_{15}^2 =$  $\subset S_{269} =$ ,   $S_{91}^2 =$  $\subset S_{270} =$ ,

$S_{25}^2 =$  $\subset S_{271} =$ ,   $S_{163}^2 =$  $\subset S_{272} =$ ,

$S_{66}^2 =$  $\subset S_{273} =$ ,   $S_{72}^2 =$  $\subset S_{274} =$ ,

$S_{50}^2 =$  $\subset S_{275} =$ ,   $S_{46}^2 =$  $\subset S_{276} =$ .