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On a metamatemEthical question in talent care*

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Abstract

Recently more and more ethical issues arise in several sciences. We think that didactics of mathematics is not an exception. In this paper we investigate the question whether we can allow from mathematical precision in talent care. We suggest that these questions origin even from the formulation of a problem. The formulation of three well-known math problem is analized.

Keywords: talent care, ethical issues, problem posing

1. Ethical questions in science

“Mathematics is useful because we can find things to do with it. With this utility ethical issues arise relating to how mathematics impacts the world. (...) We study one of the most abstract areas of human knowledge: mathematics, the pursuit of

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absolute truth. It has unquestionable authority. Indeed, it is clear that mathematics is one of the most useful and refined tools ever developed. When something is useful, however, it can often also be harmful; this can be either through deliberate misuse or ignorance." [3] Although the system of mathematical thinking is a closed system, the results of mathematics are widely used in real life. Usually, a mathematician cannot be held responsible for the applications of their mathematical findings as those theorems are purely theoretical and have a well-defined system of conditions. At the same time in real life the same findings are often applied without checking the conditions. Still, these "uncontrolled" deductions are usually true. The opposite case is also possible. For example, in modeling problems, it is almost never possible to give a precise model to the task, and very often it is also impossible to translate the practical model to a mathematical one. Applying mathematical theorems without due prudence (e.g. leaving out conditions) can cause serious problems. Take the global financial crisis of 2007–2008 as an example, as so did [3]. The causes of the GFC are complex; however, there is consensus that mathematical work played a vital role. Unfortunately, the mathematical model and pricing of Collateralised Debt Obligations were based on several assumptions some of which did not hold. In the end, it led to the write-down of $700 billion of CDO value from 2007 to 2008. The rest is history.

It is worth considering whether a mathematician should care about the aim of their mathematical task, i.e. what will their results be used for. Should they solve a problem if they know that the solution can be used to cause harm? It is not a specialty of mathematics; similar dilemmas appear in other branches of science. A classic example is that of the physicists taking part in the Manhattan plan. Their findings are revolutionary as scientific innovations, still, their work leads to the creation of a weapon capable of destroying humanity. Similar ethical dilemmas arose concerning the work of Ede Teller. Let us quote the famous scientist himself about the issue: "The scientist is not responsible for the laws of nature. It is his job to find out how these laws operate. It is the scientist's job to find the ways in which these laws can serve the human will. However, it is not the scientist's job to determine whether a hydrogen bomb should be constructed, whether it should be used, or how it should be used." He reinforces this point of view later [6] stating that the scientist's responsibility extends to work and to explain their findings along with the possible consequences – and no further.

Are any of these ethical issues relevant for a pure mathematician, say, a number theorist working in academia? Suppose they develop an algorithm for fast factorization. Should they publish it? If so, when, where, and how? If not, what should they do? Should they have thought about it beforehand? – asks Chiodo and Clifton [3]. Based on interviews, the typical answer would be that they would publish it immediately as they have the right to do so. But the consequences would be problematic – for instance, the breaking of RSA encryption in a chaotic manner could result a collapse of internet commerce and the global economy.

The following question also arises: Are there ethical dilemmas concerning the teaching of mathematics? Let us give some examples. Only a small part of the wide range and great depth of known mathematical ideas can be shown during maths lessons. The yet limited competencies of students often make giving clear definitions and exact proofs impossible. Instead of proofs, it is not rare to demonstrate only trains of thoughts. When teaching the definition of a prime number in high school, we give a definition that is mathematically incorrect. It is an important question whether pupils are deceived when they are given incomplete definitions or is they are given trains of thoughts instead of proofs. Do we do them wrong by giving a false image of mathematics and mathematical thinking? Luckily this question is already well-handled in the education of the methodology of mathematics and there is a classical saying of Éva Vásárhelyi addressing this issue: "We have to grant some mathematical inaccuracies in favour of comprehensibility due to the level of proficiency of the students" [5]. We can see many occurrences of this kind of inaccuracies in primary and secondary level mathematics education, mainly when working on developing concepts. As a concept develops, in time, it becomes clearer. For example, when introducing exponential functions, understanding precisely why they make sense is out of reach for the students. Then, most of the concepts which were initially sloppy and loose, become exact by the time of final exams. These initial inaccuracies or gracious lies are serious errors from the aspect of mathematics but they are unavoidable because of the spiral structure of the curricula (key concepts are presented repeatedly throughout the curriculum, but with deepening layers of complexity, or in different applications) [2]. However, spiral curricula are well-reasonable from a developmental cognitive psychological point of view (e.g. the information is reinforced and solidified each time the student revisits the subject matter) [2].

Probably the most obvious ethical dilemma of teaching mathematics is what should appear in the National Core Curriculum (NCC), the law which regulates the official learning material in Hungary [26]. Thus, the first question that should arise in those who are preparing the NCC is which topics and competencies to include and whether these topics and competencies reflect the mathematical education that we want to mean by mathematical education. A row of ethical questions can be posed concerning the transitions from NCC to the framework curricula, then the local curricula and the syllabus, and at last the practice of teachers. This latter one includes an already debated issue, namely that teachers tend to teach students towards the maximum percentage on the school-leaving exam by endless mechanical practicing rather than fulfilling the aims set in NCC [11].

2. Ethical dilemmas in talent care

In this paper, we focus on ethical issues concerning mathematical talent care, which has long traditions in Hungary. One of the first mathematical journals was established and published in Hungary: Arany Dániel founded the "Középiskolai Matematikai Lapok" in 1893, the first issue was published in 1894. The journal has been functioning since. In the aspect of mathematics, Hungary belongs to the elite of the world. This fact is strongly related to talent care. Children participating in talent care programs or optional math classes (e.g. after-school classes) face the concept (and challenge) of giving arguments and proofs much more than their peers. As they need to give more and more accurate proofs (on talent care lessons and competitions), they reach a deeper level of understanding in mathematics. The mathematical development of students is largely affected by problems and problem compilations posed on talent care classes. Some well-known examples of problem books are: Szakköri feladatok matematikából 7–8. osztály (Problems for special math classes grade 7-8.) [20], Szakköri füzetek – Számelmélet (Optative math booklets – Number Theory) [21], Prímszámok (Prime numbers) [19], Négyzetszámok (Perfect squares) [18], Kombinatorika (Combinatorics) [16]. These booklets are well-known and used on paper or online by many students interested in mathematics. What ethical questions can be posed concerning talent care? Can we grant mathematical inaccuracies in favour of comprehensibility even in talent care? Can we correct inaccuracies and gracious lies that have occurred in normal mathematics class? How can we communicate so that neither knowledge nor authority is hurt? We have picked one of these issues and have transformed it to the following research question: In contrast to the inaccuracy that is accepted and often necessary in regular mathematics classes, can we give an inaccurate, mathematically incorrect answer or solution to a question or problem in talent care? Some problems can be considered typical in talent care as they appear often. We deal with three branches of problems which we will name "balance scale-" "statements-" and "camel-" problems. After analyzing the problems, we will also discuss the ethical questions arising concerning them.

3. Problems in talent care

In this section, we show three families of problems appearing in talent care. Two of these problems are of current interest among mathematicians, too. We try to analyze to what extent these problems can and should be posed to high-school students.

3.1. Balance scale-problems

Consider the following problem: Given 9 coins, one of them fake and lighter, find the fake coin in two weighings on a balance scale.[17]

The official solution the booklet shows that it can be done with three weighings, and does not show that two weighings are not enough. The following problem is handled similarly:

Which of the 8 coins is the fake one? There are 8 coins; one of them is fake. All real coins weigh the same. The fake coin is either lighter or heavier than the real coins. Find the fake coin and figure out whether it is heavier or lighter than the others, in the minimum number of weighings on a balance scale. [17]

One might think that the balance-scale problems are traditional and ancient. Yet, this type of problems is quite novel. Surprisingly, its first publication was by E. D. Schell in the January 1945 issue of the American Mathematical Monthly [22]. The solution for n coins can be found for example in [24], a rather popular high-school problem book. The problem can be posed for several fake coins, and the complete solution is not known. The best known construction for n coins and unknown many fake coins has $7/11n$ many weighings [12] and the best known lower bound is [13] $\log_3(2^n + 2^{n-5} + 2^{n-6} + 2^{n-7} + 2^{n-9} + 2^{n-10} + 2^{n-12} + 2^{n-13})$. We can see from this formula that the general solution of this problem does not only look hard but is not even known. We might still think that for a small, given amount of coins we could pose the problem for students and after they tried cases and experimented, it is easier to show them that the lower and upper bounds are equal. Unfortunately, this is not a viable option either. For example, 11 and 2 are small numbers, so based on the assumption above, finding 2 fake coins out of 11 would be a problem suitable for secondary school students. In this problem, the number of the necessary weighings is 5 which was found out in 2015 and the proof uses the ternary Virtakallio–Golay code [4].

3.2. Camel-problems

We have camels and water and we want to cross a desert. The camels can carry a given amount of water and they can pass water to each other. They also consume water continually. The first question is if we have a given number of camels and all of them need to return to the starting point, except one. Then how far this exceptional camel can go. The second question is how many camels do we need if we want to get to a given distance. The following two analogous problems can be found in [17].

Peregrination in the desert. Ali ben Yusuf works far from his hometown, with a hundred-kilometer-wide desert between his workplace and his parents' house. He wants to visit his parents and starts planning the trip. It turns out that one can travel 20 km a day and the maximum weight to carry is three days' food and water. For simplicity, let us suppose that he can make dumps only after a whole day-route. How many days does he need to cross the desert?

The exact solution of the camel-problems is not known. The problems about desert-crossing were first introduced in 1947 [7]. The second version of the camel problem is formulated with jeeps instead of Yusuf, and is known under the name of the Jeep-problem. An analysis can be found in the article Gale's Round-Trip Jeep Problem [10]. The paper contains the proof of the following theorems. Let us suppose that we have enough fuel for n days. Then the maximum distance reachable with one jeep is $D_1 = 1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1}$. Numerous versions of the jeep problem were solved [1, 8, 9]. In each work it is in common that both the readers and the authors themselves have a feeling of deficiency. Although every paper solves some problems, none of them reaches the goal it aims at.

3.3. Problems about statements

Two hundred statements. "On one side of a sheet of paper the following list of statements can be found:

1. At least one of the statements on this paper is true.
2. At least two of the statements on this paper are true.
3. At least three of the statements on this paper are true.

...

99. At least ninety-nine of the statements on this paper are true.
100. At least a hundred of the statements on this paper are true.

If we turn the sheet over, the following can be read:

1. At least one of the statements on this paper is false.
2. At least two of the statements on this paper are false.
3. At least three of the statements on this paper are false.

...

99. At least ninety-nine of the statements on this paper are false.
100. At least a hundred of the statements on this paper are false.

The text is continuous, we have left out some sentences (marked by three dots). How many true statements are there on the paper?" [17]

It is easy to ascertain that all hundred statements on the first side of the paper are true and on the second side statements 1–50 are true and statements 51–100 are false. Then the answer is that there are one hundred and fifty true statements on the paper.

Eight statements. "The following statements can be read on a paper:

1. At least one of the statements on this paper is false.
2. At least two of the statements on this paper are false.
3. At least three of the statements on this paper are false.
4. At least four of the statements on this paper are false.
5. At least five of the statements on this paper are false.
6. At least six of the statements on this paper are false.
7. At least seven of the statements on this paper are false.
8. ...

Unfortunately, the eighth statement is illegible. Is statement eight true or false?" [17]

Let us examine the first statement problem for three statements. Then the statements are the following:

1. At least one of the statements on this paper is false.
2. At least two of the statements on this paper are false.
3. At least three of the statements on this paper are false.

This problem has no "solution." – the problem itself is not even a proper problem as it has no exact mathematical sense.

3.4. Students' possible dilemmas concerning the problems

The foundations of the methodology of problem-solving have been laid by György Pólya [14]. Since then problem-solving became an autonomous branch of didactics [23] with numerous aspects out of which we now highlight only one: Problem-solving as a thinking activity involves re-formulation, analysis, generalization, and extension of problems. This is how the idea of solving the three-statement-problem can appear after solving the hundred-statement-problem. Students working on generalizations of the problem probably see that they cannot give a solution for any odd number of statements. At this point they might become frustrated, think that they misunderstood something or made a mistake and start developing mathematical anxiety. They might also get confused not being able to solve a problem that is similar to one that they have already solved. They might even think that their previous solution might have been wrong.

The eight-statement problem can cause dilemmas already when interpreting the problem. One might try to analyze what happens if they write a specific sentence to the eighth place. In case of different sentences the conclusion can be different. If we write "$2+2=5$ over integers." the problem is solvable, but if we write "$2+2=4$ over integers.", we get a contradiction – this causes confusion concerning what to say about the original problem. If we write the sentence "John eats soup." as the eighth statement, the case is even more problematic: Why would seven sentences on a paper make John eat soup?

The sample solutions presented for the balance scale-problems are not complete. In both cases, a construction for finding the fake coin by a certain number of weighings is presented. But why do we solve the second problem by three weighings, not five? It is easy, the task told us to use the least possible number of measures. But can we manage to find the fake coin with two weighings? To give an exact solution, we need to show two things. First, we need to prove that less than three measures are not enough and that three measures are enough. The sample solution only shows the latter by giving a construction, it does not even mention that fewer weighings are not enough, let alone reasoning why. We can draw the conclusion that the sample solution does not answer fully the question – while making a convincing impression in students of doing so. Even some of the authors of this article were fully convinced by this impression before changing to "teacher's view" and, after careful analysis, noticing that there are mathematical, and what is more, metamathematical errors here. Let us imagine how a student might think. They know that these problems are dedicated to their age group. They try and try and can or cannot solve the problem. Probably they get a weaker result by themselves than the sample solution. Then they read it and see that it gives a thoughtful construction which suggests that it is the best possible. Talented students are likely to think that they are not entitled to judge the correctness of the book, so if they are not convinced, they blame themselves for not understanding the book which is written by clever professionals, therefore it must be perfect. This false impression is reinforced by the fact that these problem compilations contain several similar problems of one type in one block – for didactical reasons and for making a greater impression.

Usually, it is normal and is also in accord with the theory of the zone of proximal development as it divides the chains of thoughts into steps, advancing from concrete to abstract, from small to large. This structure supports that the solution of one problem gives ideas that help to solve the next problem without taking away the joy of challenge and while providing a good experience, improving thinking as well.

The problem is that here a partial proof of the sample solution makes the impression of being a full one. These dilemmas and similar ones can appear in the case of the camel-problems, too.

4. Ethical dilemmas in talent care

The dilemmas appearing concerning the balance scale, the camel and the statement problems can be of different types: mathematical, teacher's, author's, and poser's dilemmas.

Mathematical dilemmas can be: What are the exact meanings of these problems? What are their exact solutions? Are they at all solvable? Are the conditions unambiguous? Does the problem use mathematical terms correctly? Are we sure that we do not try to see information in the text of the problem that actually is not included?

Teachers' dilemmas can be: Are we able to solve the problem? Are we able to solve with secondary-school methods? When we read the sample solution, we might also think that it is correct. But can we decide whether the sample solutions are real solutions to the problem? We must be careful when posing the problem so that we do not leave any questions unanswered or make any student frustrated. Finding a construction in case of the camel and the balance scale problems is already an exciting problem. Can we pose the problem in order to show a nice construction and at the same time without giving a full solution? If we only ask students to solve a balance scale-problem with a particular number of weighings, then we pose the problem ethically but less elegantly. But we do not address the inaccessible question: "is it possible with fewer measures?". Here the chance of students trying to generalize the problem and find the minimum number of measures still exists. We have already seen that this requires strong mathematical background. At this point we, as teachers, have an important task: we have to tell our students that there exists a minimal number of weighings, but in a lot of cases the full clarification would need a stronger mathematical background than they have. This raises even more questions: Should we show our students the solution even if we know that they will not understand everything? This way we make them see that the problem is solvable. We can also tell them that similar problems are subjects of great interest among mathematicians, too. We can mention that some of the problems are already solved but some of them are still unsolved. We can also give information on the specific problem we posed: who solved it and when.

Problem posers' first dilemma is how to pose these problems: To which age group can we show the problem? To which age group can we pose the problem so that we can tell the solution, too? How much experience do students have with making proofs? Will they feel the need for proof? For those who feel, we might cause momentary frustration (if they do not have the competence to give correct proof). For those who do not realize the need for proof, we do not cause frustration. Their problem can appear later on when they see other types of thoughts and think that they are proofs. Another question is whether to pose only the easily solvable part of the problem. Here the dilemmas are similar as in case of teachers.

Author's dilemmas appear when someone starts to write a book or a compilation of problems. The first "author's dilemma" is whether I can include a problem in my book knowing that I must provide an incorrect or incomplete solution. We can see a lot of examples of this in books. The exact solutions of these problems cannot be presented in books for primary or secondary school students as they require higher mathematical knowledge. Another possibility is posing only a part of the problem. This makes it less appealing, maybe it will not even fit into the book. But if I omit it, I might deprive the readers of getting to know a nice, deep thought.

5. Interviews

To resolve as many dilemmas as possible, we conducted two interviews. One with PhD students to see how they solve the problems analyzed above. Are the solutions of these problems adequate to discuss on PhD level? Do doctoral students need directing questions to answer their own arising questions?

For the second interview we asked Sándor Róka, the author of [16–18] and many other problem books. He is one of the outstanding characters in Hungarian mathematical talent care. The idea of conducting an interview with him is thrilling and frightening at the same time. He gladly answered our questions and even after the interview he continued to share his thoughts with us via email. These thoughts are based on decades of experience in leading talent care courses for students from different age groups, starting with primary school, up until university level. Among his several widely used booklets and books, probably the most well-known is "2000 problems from the field of elementary mathematics" [15] which is part of the recommended literature for pre-service teachers. Sándor Róka was an educator in teacher training for a long time at the University of Nyíregyháza, now he focuses on talent care for upper primary and secondary students, mainly within the Erdős Pál Talent Care Center.

5.1. Interview with PhD students

To disclose the dilemmas concerning these problems and reveal the mathematical disorders in them we conducted an interview. The interviewees were three PhD students, all of them having a master's degree in mathematics education and participating in the Didactics of Mathematics PhD-program. Besides their PhD studies and research, they also teach mathematics at primary or secondary school. The interview consisted of open questions in connection with five tasks. The five tasks were consecutive problems from [17]. The whole interview was videotaped. In the structure of the interview, the consecutiveness of the five tasks and the fact that each problem is built (in some sense) on the previous ones held a key role. At the beginning of the interview the interviewees were asked to analyze the problems one by one along with their solutions both from a student's and a teacher's point of view, parallelly. We summarize the interview. If necessary, we quote participants, they will be denoted by A (interviewer), and P_1, P_2, P_3 the PhD students.

The first two problems in the interview were the "Ninety-nine statements" and the "One hundred statements" problems.

Ninety-nine statements. The following statements are written on a paper:

1. Exactly one statement is true on this paper.
2. Exactly two statements are true on this paper.
3. Exactly three statements are true on this paper.

...

99. Exactly ninety-nine statements are true on this paper.

The text is continuous, we have left out some sentences (marked by three dots). Find out which statements on the paper are true.

One hundred statements. The following statements are written on a paper:

1. Exactly one statement is false on this paper.
2. Exactly two statements are false on this paper.

...

99. Exactly ninety-nine statements are false on this paper.
100. Exactly one hundred statements are false on this paper.

The text is continuous, we have left out some sentences (marked by three dots). Find out which statements on the paper are true.

The students solved the first two problems without spending too much time. The third problem of the interview was the "Two hundred statements" problem from Section 3.3.

At this problem, the answer is not so obvious. After considering the possibilities a linear ordering of the statements was proposed.

P1: "From at least x true statement, at least $x - 1$ is implied."

Then "open problem", doubts arose considering about what do we call a statement:

P1: "It might be an open problem with multiple solutions. The definition of a statement is: a sentence is a statement if it is clearly decidable whether or not it is true."

P2: „The text of the problem tells that they are statements so we cannot say that they are not... or that the poser of the problem is not right."

After thinking a short while, they proved that the statements on the first page must be true, and they started to think about how many false statements have to be on the second page to get an adequate number of false statements. Then they quickly finished the analysis and the solution of the problem. The last two problems were "The mysterious rock" and "The eight statements" problem from Section 3.3.

The mysterious rock. Once upon a time, two kings were fighting against each other. When one of them won and occupied the other's castle, he found a strange rock in the castle yard. On the top side of the rock, there was the exact same statement engraved 77 times: "There are at least 77 false sentences engraved on this rock." Next to the rock, there was a small table with the following explanation: "On the bottom side of the rock there are as many statements as on the top side, but these statements cannot be seen by any human." How many true statements are there on the rock?

In the first part of their search for a solution they reached again the question of what we call a statement in mathematics. They started to understand that they had not considered the exact definition of a statement.

P1: "Because of the other statements it is necessary that statement 8 is false. This way, there will not be any contradiction in the system."

P2: "It should be stated that the problem has a solution. Because ... for example, let us take "The sky is blue" as statement 8. Will "The sky is blue" be false because it should be false based on the other statements?"

P1: "Of course not ... We only have to decide, whether or not the 8 statement on the paper was true."

A: "What is the matter with the 8 statements problem? I mean, the main problem... didactically and mathematically."

P2: "Didactically the main error that it is like the task: Follow the sequence: 5, 10, 15, 20," (meaning, that these kind of problems are not well defined, you need to figure out, what the problem poser thought.)

A: "What is the mathematical error??"

P1: "What is a statement?"

After discussing and clarifying the notion of statement, the next dilemma arose:

P2: "Is it not the resolution that we know from the formulation of the problem that these sentences are statements?"

At some point the students reached the conclusion that in these problems there is no given frame system (axioms) based on which we could decide whether these statements can be formally deduced or not. Firstly, there is no way to give true or false values to these statements such that they become consistent in the classical human language. Secondly, there is no base of knowledge that would tell us which sentences are statements. Thirdly, students, especially high-school students are not supposed to be aware of these ideas. So it is a kind of cheating not to tell students that here we have (or might have) paradoxes.

During the interview the three PhD students wanted to solve the problems in the first place instead of interpreting them. Right after the beginning, although P2 mentions that they should analyze the notion of statement, they soon got back to searching for the solution. The timespan of the interview was nearly an hour and it was only towards the end when PhD students started to have doubts about the sense of the problems – or we should rather say the senselessness. None of them could tell the correct, exact definition of a statement.

One needs a very strong mathematical background and intelligence to start having doubts concerning the problem itself and the way of its posing. At PhD level this is attainable. The interview shows that the clarification of the problem at secondary school level would be very hard. A nice example of the resolution of a contradictory problem is the next one about knights and knaves. "Suppose A says, 'Either I am a knave or else two plus two equals five.' What would you conclude?" [25] The official solution presented in the book is the following: "The only valid conclusion is that the author of this problem is not a knight. The fact is that neither a knight nor a knave could possibly make such a statement. If A were a knight, then the statement that either A is a knave or that two plus two equals five would be false, since it is neither the case that A is a knave nor that two plus two equals five. Thus A, a knight, would have made a false statement, which is impossible. On the other hand, if A were a knave, then the statement that either A is a knave or that two plus two equals five would be true since the first clause that A is a knave is true. Thus A, a knave, would have made a true statement, which is equally impossible. Therefore the conditions of the problem are contradictory (...) Therefore, I, the author of the problem, was either mistaken or lying. I can assure you I wasn't mistaken. Hence it follows that I am not a knight. For the sake of the records, I would like to testify that I have told the truth at least once in my life, hence I am not a knave either." [25] So it can mean a resolution if we admit that the problem-poser either made a mistake or lied.

5.2. Interview with Sándor Róka

When preparing for the interview we chose to focus our questions on his problem compilations and beliefs and principles as a problem poser. He was very open towards us and started sharing his views as a problem poser and talent nurturer almost without having to ask concrete questions. The online interview lasted for 80 minutes. We quote some highlights from the interview that are interesting to our paper.

A: I found several problems in your books where the solution is practically a construction (e.g. the balance-scale problems when we have to find a fake coin). In this case we provide some inaccuracy, since with these constructions we can only show that a certain number of weighings is enough. What is your opinion in general about providing inaccuracies and demanding less mathematical preciseness even in talent care in order to make the problem more understandable?

RS: We do not look mathematics as "definition, theorem, proof, and then everything is complete". This is not the way mathematics was explored. This would be an exaggeration. Precise axiomatic mathetatics is important only for very few people. I am not sure that it should be introduced to children. They are not really interested in this more abstract part, they don't understand why it is important. But problems asking for constructions appear a lot of times in competitions. Several times both directions are asked. If they ask to demonstrate that you can solve the problem with a given amount of weighings – you just show a construction. In case of a lot of balance-scale problems you can argue that fewer weighings are not enough, so our construction reaches the optimum. Of course, this latter problem can be incredibly hard, I know. For example, take the problem of finding the two heaviest among 8 different balls – telling how many weighings we need is hard. But it is clear that to find the heaviest one we need seven weighings, we can give reasoning and also show a construction how to do it.

A: Do you think that there is a level in talent care when (or starting from when) things can be clarified? Do you have a solid opinion on who should support students in this clarification if later they feel the need to make everything precise? Whose task or role is to clarify the problems already seen without losing preciseness?

RS: I think these questions only bother specialists and maybe very few "gourmands" ...

A: As you have already mentioned there are problems which can be approached from several aspects and can make the solver think about numerous related problems. To what extent is the phrasing of the problem important in this case? As you have already mentioned, the way how we pose the problems can be important. For example, take a "crossing a desert"-type problem. We can ask "how far can XY go" or we can ask "how XY can reach the furthest possible". According to you, what is or what can be the role of phrasing?

RS: In this certain case it is quite random which option of phrasing I choose – sometimes this, sometimes that. But if I ask "which far can XY go" – then when someone tells a numerical answer, they also have to explain how that is possible... I guess the answers to these two questions somehow go together. But, surely, the way I ask questions, the way I compose them is important.

During the interview Sándor Róka made clear that it was an important aim of all problems in his books to raise students' interest. It is important that the phrasing is catchy, so students start to solve the problem because of its exciting and interesting nature. In talent care classes it is the problem poser's task to care with the problem profoundly and give as precise solution as possible. Earlier when dealing with balance scale-problems, Sándor Róka himself also prioritized preciseness so he asked for example "Can you solve with five weighings?". Nowadays, he rather tells them to try to solve with as few weighings as possible. This way he can achieve that everyone can feel themselves successful – even those who did not find the optimum solution. During the discussion students see that there are several ways to solve the problem and there might be more effective solutions (in this case, with fewer weighings). With the most interested students the professor addresses a problem from several aspects and with several questions. During this process the details that were inaccurate at the beginning can be clarified. This clarification is the task of the teacher, along with giving answers to the questions that arise about the problems. This spirit appears in his books also.

6. Resolution and summary

In our paper we addressed an ethical question related to mathematics education. We analyzed whether we can give an inaccurate, mathematically incorrect answer or solution to a question or problem in talent care in contrast to the inaccuracy that is accepted and often necessary in regular mathematics classes. While inaccuracies and white lies are normal in regular math lessons as part of the spiral method of concept building, are they acceptable in talent care, too? When analyzing the statement-, balance scale- and camel problems, two types of questions arose. The first question is whether it is acceptable – and if yes, then in what content – to play with concepts that students do not know exactly. Distinguishing between the everyday sense and the mathematical concepts of a statement is not easy. The difficulty of the other two types of problems comes from the difference between a construction and an extremum. In both cases it is a challenge for a student to find an optimal construction, but giving a proof that the construction is optimal is beyond a secondary school student's competence. Concerning posing and presenting of problems, several dilemmas and metamathemEtical questions arise. After analyzing these questions we presented the extract of two interviews. In the first one participants are PhD students and the interview focuses on the mathematical background of a problem. In the second one the interviewee is Sándor Róka, one of the most well-known problem creators in Hungary, author of multiple books and problem booklets. In the first interview it turned out that the solution and resolution of the problems is a challenge even for PhD students. It requires serious mathematical background and intelligence to doubt the posing of the problem itself. This interview enlightened that clarifying these problems would be very difficult at secondary school level. The short conclusion of the second interview is that a problem poser needs to pay attention to a lot of aspects at once. The borderline of these aspects is sharp and we have to cross at least some of them. According to the Ars Poetica of Sándor Róka, the first border to keep us inside is that of the attention and the interest of students. Without the mathematical commitment of the students, without gaining their attention and raising their interest none of the further questions can even appear. Later, when students and their teacher deal with the problems together and some doubts arise at border-crossings, they need to resolve the doubts together.

Talent care is a significant task. It requires profound planning, a lot of preparation, and continuous attention.

When Ptolemaios the first asked Euclid whether there is a way of learning geometry that is easier and shorter than the one presented in Elements, the great geometer answered: There is no royal road to geometry. In the footsteps of Euclid, we can say: There is no royal road to talent care.

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