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The education and development of mathematical space concept and space representation through fine arts

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Abstract

The aim of this paper is to present potential connection points and interrelations between the development of spatial abilities and concepts and fine arts in high school (K9–12) education. The evaluation and comparison results of the assessment of spatial abilities and freehand drawing abilities show strong correlation, which fact further supports this potential educational interconnection.

Keywords: spatial abilities, teaching of fine arts, visualisation of space

MSC: 97D40, 00A35, 97D60

1. Introduction

Development of spatial abilities and concepts is of utmost importance in school education. Many papers studied the importance of spatial visualisation skills in various countries and in various school levels (for a good overview see e.g. [1, 6]). Spatial skills and spatial visualisation abilities have no unique definition – here we apply the following concept: by spatial ability we mean the ability of perception of two and three dimensional shapes, and the ability of application of the perception of these shapes and their relationships in spatial reasoning and solving spatial problems [10].

Spatial abilities or some of their aspects can be assessed through various tests, among which one can find international standards (such as Mental Cutting Test, Mental Rotation Test, see [3, 4, 12, 13]) and national versions as well. Most of them are specified for a certain aspect of school period/age. Several studies, including Hungarian surveys, observed gender differences in these abilities, which can and must be considered during the educational process [6–8].

It is also known that spatial ability is not the result of a person's ability to be born with it, but is the result of a long-term learning process([10, pp. 40–45]). The lack, or rather deficiency, of spatial ability yields problems only in a rather narrow part of the mathematics curriculum (spatial geometry), but in practical terms, in everyday life, correct spatial approach is extremely important and should be improved.

According to psychological studies, the development of visual thinking is completed relatively early. Therefore, neglecting spatial geometric problems and developing spatial attitudes in elementary school at the age of 10–14 can lead to an imminent disadvantage [9]. Literature data (see [10] and references therein) prove that the spatial concept of students can still be improved at the age of 12–16, but only to a certain extent. That is, in high school, spatial awareness is more difficult to develop, especially if a pupil comes from the primary school already possessing spatial deficiencies.

Therefore, we must grab all the tools, fields, subjects and lessons that can help us further develop spatial abilities. Beside mathematics lessons, which are natural milieu of development, other study fields can also support this development.

In this paper we discuss the correlation between art history and spatial visualization in order to show potential connection points in terms of development of space concept and space representation through fine arts. We intend to specify those periods or styles in the history of art, which can be assigned to well defined spatial visualisation problems. This is discussed in Section 2. In Section 3 we present outcomes of a test period, where spatial ability tests and drawing tasks have been tested and their correlation has been studied. This study supports our initial hypothesis, that there is a strong correlation between spatial abilities and (spatial) drawing abilities, which makes sense the above mentioned interdisciplinary approach.

2. Periods of art history and spatial visualisation

Problem solving on the plane of a drawing sheet is evidently not sufficient to develop spatial concept, to improve the spatial abilities. It is also important to act and construct in space for better understanding and more effective development. Kárpáti et al. found that the most effective developmental procedures are real-world operations: making sculptures and installations, modeling, object creation. ([5, p. 103]).

Where else can students find and study spatial constructions and their representation? Evidently in lessons on fine arts. Since people exists in space and time, from the beginning of (art) history man is deeply interested in the visible and tactile space, and its visual expression, the planar representation of space. At first we briefly review the development of spatial approach and spatial representation, as well as its development in different eras of art from this point of view.

Period/style of art	Topics to study	Spatial visualisation
Prehistoric and Egyptian Art	Cave paintings (side view), Egyptian paintings (principle of largest surface view)	Similarity to orthogonal mappings
Art of the Roman Empire	wall-paintings (Pompei) and mosaics	Analogue of axonometric mapping
Medieval Art	Byzantine icons, codex illustrations	Reverse perspective
Late Medieval Paintings	Giotto frescos	Similarity to axonometric mapping
Renaissance	Development of correct perspective drawing through many artworks of various artists	Perspective mapping
Baroque, Classicist and Romantic Paintings	Illusionistic ceiling paintings, flourishing of perspectivity	Perspective mapping
Impressionism, Post-impressionism, Cubism	Monet, Cézanne, Gauguin, Picasso, Braque, emphasising the cubist approach (Braque, Picasso), where spatial relations and structure of objects have been studied, with manifold unified views and mappings in the same figure. Mondrian's geometric compositions	Different parallel projections
Op-art, Contemporary Graphic Art	Art of geometric compositions (e.g. Victor Vasarely and Maurits Escher)	Non-linear mappings

Table 1: Embedding spatial visualisation methods into teaching of history of fine arts

Here we only review and list a few important elements of potential areas of drawing lessons in the National Core Curriculum, where spatial ability can be intentionally developed. For a more detailed overview see [10, pp. 135–148].

2.1. Primary school

Class 5: Making apparent drawings depicting spatial situations after a sight and based on imagination. View and derivation of objects from simple geometric shapes. Output: draws a fictional object based on memory, imagination. Knows how to display spatial situations. Class 6: Preparation of shape analysis structural drawings, sections, reductions. Projection representation. Output: Apply the familiar representation modes as appropriate. Class 7: Means of plastic expression (degree of spatial extent, directions, articulation, place in the environment). Structural, perspective representation of larger artificial and natural forms, Monge projection, (one-dimensional) editing of axonometry. Designing a utility object by making an appearance and projection drawing. Output: Its ability to abstraction is manifested in the emphasis on substance, in geometric simplification. He knows the basics of Monge projection and axonometric representation, he/she solves such an editing task with more or less independence. Class 8: Observation of a built spatial unit (building, street detail), experience-perspective representation. Edit a one- or two-way perspective image. Use of longitudinal and cross-sections to make illustrative diagrams. Reconstructions based on projection and axonometric drawings. Observation of the representation that creates the illusion of space, apparent shortenings, point of view. The perspective representation. Spatial representation modes, mixed perspective representation modes in different ages. Output: Has the necessary spatial basis for representation conventions, is able to edit simple projection, axonometric and perspective figures.

In grade 7–8. it is no accident that the curriculum related to the spatial approach swells in the classroom. These are the two school years when students who are no longer in secondary school must be taught all this. Another question is how much the number of hours shrunk to one drawing lesson per week is enough to master the diverse curriculum [10].

2.2. Secondary school

Prior to the new framework curriculum introduced from the 2014/15 school year, in secondary school (in vocational high school from the 2016/17 school year), it was possible to learn more about spatial representations and apply them to students as part of a drawing and visual culture class. There was an opportunity for freehand drawing and even occasionally even editing. The development of spatial representation can be traced throughout art history, which could be interestingly approached and skilfully emphasized in drawing lessons on art history topics. (As a drawing teacher, I also used it in secondary school for as long as I could – in the framework of the Drawing and Visual Culture subject, before the new framework curriculum introduced from the 2014/15 school year.)

The way in which space is depicted is a central issue in painting, as it also expresses the painter's relationship to reality, and the worldview of the ages can also be read from it. (Thus, from prehistory to the renaissance, where painters had perfected the experiential perspective – which mathematicians only wrote much later – we could arrive at isms, modern endeavors, where they again began to ignore perspective.)

3. Experimental study of the relation between spatial abilities and art

In order to apply the above principles, we need to test that drawing skill and spatial ability are related. Therefore we organised an experimental study in this regard. 9th and 12th grade students participated in the survey. The hypotheses are that the above mentioned correlation exists, that final year students score higher (perform better) on the spatial test than 9th graders, meaning students increase their spatial performance with age, and there is a clear gender difference in these abilities. Continuing the previous studies in gender differences, and applying their methods ([6–8, 10]) we also studied this latter aspect. While previous studies mainly focused on university students, here we prove through experimental data that there is a difference in the spatial and performance of boys and girls already in high school. According to the results of an experiment([2]), the development of the spatial ability is significantly reflected by the students marks in maths and drawing. Our main focus is on the correlation between drawing abilities and spatial abilities.

We present the evaluation of the first three tasks of the written test series, which require mathematical and geometric knowledge in addition to the appropriate spatial approach, in terms of the two age groups (grades 9 and 12), and gender (girls and boys). We also examine the relationship between test scores and students' representation of space from memory. Drawing from memory is more difficult than drawing from sight, because from memory one draws what one knows about objects – based on schemes. In the case of a room and an interior, on the other hand, the task is to display the space, to place and draw the objects in space (abstracting from schemes, application of a representation system is necessary).

So this work raises the following questions:

- How did students of different ages pass the spatial test?
- How did students of different ages perform in the part of the test that also required knowledge of mathematics?
- Is there a significant relationship between students' performance in the test and the spatial quality of the room drawings made from memory?

4. Test results and correlations

The survey was conducted in two vocational grammar schools of a Transdanubian county seat, among "incoming" (9th grade), starting the vocational grammar school, and graduating (12th grade) students. Regarding the qualification in drawing, it should be mentioned that the graduates had a Visual Culture class in the 10th grade, 1 hour per week, and one class (21 people – 12 girls and 9 boys) had a Technical Representation class in the 9th grade, for one semester, 1 hour a week. Incoming 9th grade students are equipped with the drawing knowledge and skills learned in primary school (they will not have a Visual Culture, or Technical Representation, or any other "drawing" class during their high school years). The test is based on the tasks from the online database [11].

The tests were written at the beginning of the school year – in the second half of September. The drawing assignment was completed a few months later, in November and December. Due to this time delay, a few drawings may not have been drawn, or conversely, some drawings may not be assigned to test. The number of completed drawings became less than the number of test writers, so we take the number of written tests as a basis, and we have associated the drawings with these tests. For those tests no drawing was assigned, the category "none (drawing)" is created.

The test was written by: 295 people (241 girls and 54 boys); the drawing task was completed by 262 people (214 girls and 48 boys) in 12 classes. (no drawing: 27 girls, 6 boys) In grade 9, the test was written by 182 people (153 girls and 29 boys); the drawing task was completed by 163 people (137 girls and 26 boys) in 6 classes. (no drawing: 16 girls, 3 boys). In grade 12, 113 people (88 girls and 25 boys) completed the test; the drawing task was completed by 99 people (77 girls and 22 boys) in 6 classes (no drawing: 11 girls, 3 boys).

Tasks of the test include standard spatial problems about a cube. For example, we have marked some points on the edges of a cube (see Fig. 1).

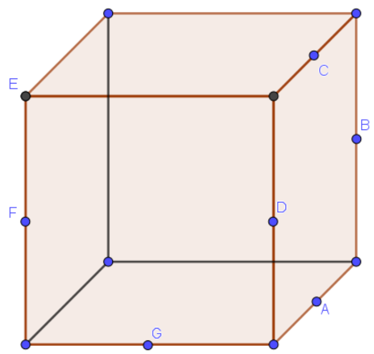

Figure 1: A cube with edge mid-points for spatial tasks

Students have to find a) four-point sets that are coplanar, and b) four-point sets that are not coplanar. A total of 10 points can be obtained in the task, from which 4 points could be obtained with four good solutions in part a). There are many good solutions in part b), where 1 point is awarded for a good answer, but we maximized the points that can be obtained in 6 points.

Another task is to determine what type of triangle are defined by the point triples BCD; BFD; FGC; ACE; GBE. A total of 10 points can be obtained in the task. Since triangles can be grouped according to their sides and angles (6th grade curriculum), the triangles listed by the points (their vertices) must be given in both ways.

In the spatial tasks the usual gender difference is observed, as one can see in the Table 2.

	9th grade average score	12th grade average score
overall	3.63	4.64
girls	3.48	4.28
boys	4.38	5.88

Table 2

We can also observe a significant improvement from 9th to 12th grade. More precisely, testing the hypothesis that this improvement is significant, the result of the t-probe is 0.0024 overall, 0.0291 for girls, and 0.0432 for boys, all less than 0.05.

What is even more interesting is our second hypothesis: that the spatial abilities measured by the test and the freehand drawing abilities measured by a drawing task (see below) are in correlation.

The drawing task had to be done freehand, that is without the use of a ruler, compass or other tool. The time allotted for the drawing task was 20 minutes – not much, but enough to sketch the main fixtures (optimally). Task text: "Draw your room from memory, as if you were standing in the doorway or sitting on your bed! You have 20 minutes for the task, you don't have to tone or color, line drawing is enough. (But whoever has the time and ambition, and of course the means, can also tone and color.) Write your name, year of birth and class on the back of the drawing! Use paper of size A4, preferably draw with a pencil (so that you can adjust or correct it)."

For the evaluation and classification of the drawings, we intended to define the categories in a similar way as the development of spatial representation can be traced in art history. It is surpirsing that axonometric drawings were not used, that is the expected, typically axonometric representation was not typical at all.

The established categories according to the spatial representation and the number (and proportion) of drawings corresponding to the categories by grades:

category	9th grade	12th grade
perspective (at least 90%)	4 (2.20%)	7 (6.20%)
perspective-axonometric (around 50%–50%)	22 (12.09%)	25 (22.12%)
space-like with planar elements	32 (17.58%)	25 (22.12%)
planar-like with a single view	43 (23.63%)	8 (7.08%)
mixed view with “unfolded” parts	44 (24.18%)	19 (16.81%)
simple layout (floor plan)	18 (9.90%)	15 (13.27%)
no drawing	19 (10.44%)	14 (12.39%)

Table 3

What we observed through calculating the significance is that the quality (category) of the freehand drawing and the scoring level of the test are in strong correlation in both grades. The correlation coefficient is 0.3994 for 9th grade students and 0.2937 for 12th grade students, both are well above the signifcance level of 98%, which is 0.2301.

5. Conclusion and future work

Our aim was to test a hypothesis in terms of spatial abilities of secondary school students, namely that the spatial ability, tested by standard tasks, and the freehand drawing ability, tested by a drawing task, are in strong correlation. This hypothesis has been proved, and, as a side effect, we also observed a gender difference, reported by many other publication, in spatial ability.

Our further goal is to prove the assumption that due to changes in curricula (visual culture instead of drawing and visual culture) and omission of subjects (cessation of drawing lessons), tests written in later school years show worse results, and spatial representation reaches a lower level. Further tests and drawings are needed to investigate this issue.

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