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On the k-reversibility of finite automata

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Abstract

It is a famous result of Angluin (1982 [1]) that there exists a time polynomial and space linear algorithm to identify the canonical automata of k-reversible languages by using characteristic sample sets. This result has several applications. In this paper we characterise the class of all automata for which her method is not applicable. In particular, the aim of this paper is to characterise the family of finite automata which are not k-reversible for any non-negative integer k.

Keywords: finite automata, k-reversible automata

MSC: 68Q45, 68T50

1. Introduction

Without any doubt, there is no formal model that can capture all aspects of human learning. Nevertheless, the overall aim of researchers working in algorithmic learning theory has been to gain a better understanding of what learning really is. Several models are on the basis of the so-called learning autamata. Learning automata has a wide field of applications ranging over robotics and control systems, pattern recognition, computational linguistics, computational biology, data compression, data mining, etc. (see [5], for an excellent survey). Recently, learning techniques have also become popular in the area of automatic verification. They have been used [8] for minimizing (partially) specified systems and for model checking black-box systems, proved helpful in compositional model checking and in regular model checking. The general goal of learning algorithms employed in verification is to identify a machine, usually of minimal size, that conforms with an a priori fixed set of strings or a given machine. Nearly all algorithms learn deterministic finite-state automata (DFA) or deterministic finite-state machines (Mealy-/Moore machines), as the class of DFA has preferable properties in the setting of learning. For every regular language, there is a unique minimal DFA accepting it [6], which can be characterized by Nerode's right congruence [10, 9]. This characterization is at the base of most learning algorithms [5].

It is a famous result of Angluin [1] that there exists a time polynomial and space linear algorithm to identify the canonical automata of k-reversible languages by using characteristic sample sets. This result has various applications. (For example, the song learning of birds has similarity to the grammatical inference from positive samples [13] which works as Angluin's algorithm. Certain linguistic subsystems may also well be learnable by inductive inference method [12]. Her method is applicable in the natural language processing, too [4]).

The aim of this paper is to show the limitations of her method. In particular, we characterise the class of all automata which are not k-reversible for any non-negative integer k. The author did not find any paper studying or characterising the class of automata having this property. In other words, it has a high likehood that there are no related works regarding our results.

2. Preliminaries

We start with some standard concepts and notations. All concepts not defined here can be found in [3, 6].

By an *automaton* we mean a finite Rabin-Scott automaton, i.e. a deterministic finite initial automaton without outputs supplied by a set of final states which is a subset of the state set. In more details, an automaton is an algebraic structure $\mathcal{A} = (A, a_0, A_F, \Sigma, \delta)$ consisting of the nonempty and finite *state set* A, the nonempty and finite *input set* Σ, a *transition function* $\delta : A \times \Sigma \to A$, the initial state $a_0 \in A$ and the (not necessarily nonempty) set $A_F \subseteq A$ of final states.

It is understood that δ is extended to $\delta^* : A \times X^* \to A$ with $\delta^*(a, \lambda) = a,\ \delta^*(a, xq) = \delta(a, x)\delta^*(\delta(a, x), q), a \in A, x \in \Sigma, q \in \Sigma^*$. In other words, $\delta^*(a, \lambda) = a$ and for every nonempty input word $x_1x_2 \cdots x_s \in \Sigma^+$ (where $x_1, x_2, \ldots, x_s \in \Sigma$) there are $a_1, \ldots, a_s \in A$ with $\delta(a, x_1) = a_1, \delta(a_1, x_2) = a_2, \ldots, \delta(a_{s-1}, x_s) = a_s$ such that $\delta^*(a, x_1 \cdots x_s) = a_1 \cdots a_s$.

Moreover, for every $a \in A, w \in \Sigma^*$, denote by $a \cdot w$ the last letter of $\delta^*(a, w)$. The concept of *acceptor* is a natural generalization of the concept of automaton. By an *acceptor* we mean a system $\mathbf{A} = (A, I, F, \Sigma, \delta)$ such that A is a finite (not necessarily nonempty) set, the set of *states,* $I \subseteq A$ is the set of *initial states,* $F \subseteq A$ is the set of *final* or *accepting states* and $\delta : A \times \Sigma \to 2^A$ is the *transition function.* $\mathbf{A}$ is called *deterministic* if $|I| \leqslant 1$ and for every $a \in A, x \in X$, $|\delta(a, x)| \leqslant 1$. Thus an automaton can be considered as a special deterministic acceptor. The *reverse* of an acceptor $\mathbf{A} = (A, I, F, \Sigma, \delta)$ is the acceptor $\mathbf{A}^r = (A, F, I, \Sigma, \delta^r)$ having $\delta^r(a, x) = \{b \in A \mid a \in \delta(b, x)\}$ for all $a \in A, x \in \Sigma$. An acceptor $\mathbf{A}$ is called *zero reversible* if both of $\mathbf{A}$ and $\mathbf{A}^r$ are deterministic. $\mathbf{A}$ is k-reversible for a positive integer k if $\mathbf{A}$ is deterministic, moreover, for any pair $a_1, a_2 \in A, a_1 \neq a_2$, if $a_1, a_2 \in F$ or $a_1, a_2 \in \delta^r(a, x)$ for some $a \in A$ and $x \in \Sigma$, then for every $w \in \Sigma^*, |w| = k$, at least one of $\delta^r(a_1, w), \delta^r(a_2, w)$ should be $\emptyset$. It is said that the acceptor $\mathbf{A}$ *accepts the empty word* if there exists an $a \in I$ with $a \in F$. Furthermore, we say that $\mathbf{A}$ accepts a nonempty word $x_1 \cdots x_s \in \Sigma^+$ $(x_1, \ldots, x_s \in \Sigma)$ if there are $a_1, \ldots, a_{s+1} \in A$ with $a_1 \in I, a_{s+1} \in F$, and $a_{i+1} \in \delta(a_i, x_i), i = 1, \ldots, s$. The language $L_{\mathbf{A}} \subseteq \Sigma^*$ consisting of all words in Σ^* accepted by $\mathbf{A}$ is called the language *accepted by* $\boldsymbol{A}$. A language $L \subseteq \Sigma^*$ is said to be k-reversible for some nonnegative integer k, if there exists a k-reversible acceptor $\mathbf{A}$ with $L = L_{\mathbf{A}}$. A deterministic acceptor $\mathbf{A} = (A, I, F, \Sigma, \delta_{\mathbf{A}})$ with $|I| = 1$ and $\forall a \in A, x \in \Sigma : |\delta_{\mathbf{A}}(a, x)| = 1$ can be considered as the automaton $\mathcal{A} = (A, a_0, A_F, \Sigma, \delta_{\mathcal{A}})$ with $\{a_0\} = I, A_F = F$, $\forall a \in A, x \in \Sigma : \{\delta_{\mathcal{A}}(a, x)\} = \delta_{\mathbf{A}}(a, x)$ and vice versa. Thus we can extend the concept of k-reversibility to automata in a natural way.

3. Results

The following statement can be derived directly from the definition of k-reversibility of automata (with the notations $a = a_1, b = a_2, u = w, c = \delta^r(a_1, w), d = \delta^r(a_2, w)$).

Lemma 3.1. *Given a nonnegative integer k, the automaton $\mathcal{A} = (A, a_0, A_F, \Sigma, \delta)$ is k-reversible if and only if for every distinct $a, b \in A$, there do not exist $c, d \in A$, $u \in \Sigma^*$ with $|u| = k$, having $c \cdot u = a, d \cdot u = b$ whenever $a, b \in A_F$ or $\delta(a, x) = \delta(b, x)$ for some $x \in \Sigma$.*

Next, we prove the following Theorem:

Theorem 3.2. *Let $\mathcal{A} = (A, a_0, A_F, \Sigma, \delta)$ be an arbitrary automaton. There does not exist a nonnegative integer k for which $\mathcal{A}$ is k-reversible if and only if there are distinct states $a, b \in A$, a nonempty input word $u \in \Sigma^+$, an input word $v \in \Sigma^*$, such that $a \cdot u = a, b \cdot u = b, a \cdot v \neq b \cdot v$, and either $a \cdot v, b \cdot v \in A_F$ or $a \cdot vx = b \cdot vx$ for some $x \in \Sigma$.*

Proof. First, we suppose that there are distinct states $a, b \in A$, a nonempty input word $u \in \Sigma^+$, an input word $v \in \Sigma^*$ such that $a \cdot u = a, b \cdot u = b, a \cdot v \neq b \cdot v$, and either $a \cdot v, b \cdot v \in A_F$ or $a \cdot vx = b \cdot vx$ for some $x \in \Sigma$. Assume that, contrary of our statement, $\mathcal{A}$ is k-reversible for some nonnegative integer k. By $a \cdot u = a, b \cdot u = b, a \neq b, u \neq \lambda$ and Lemma 3.1, this is impossible if $a, b \in A_F$. Therefore, at least one of a and b should be a non-final state. Thus, by our conditions, there is an $x \in \Sigma$ with $a \cdot vx = b \cdot vx$. On the other hand, by $a \cdot vx = b \cdot vx$, it is clear that $k > 0$. Now, let $k > 0$ and consider the minimal nonnegative integer ℓ with $|u^\ell v| \geqslant k$. First, we prove that for every prefix w of $u^\ell v$, $a \cdot w \neq b \cdot w$. If $u = wz$ for some $z \in \Sigma^*$, then $a \cdot w = b \cdot w$ implies $\delta(a \cdot wz) = \delta(b \cdot wz)$ which leads to $(a =)\ a \cdot u = b \cdot u\ (= b)$, which is a contradiction. Now, let i, j be nonnegative integers such that $w = u^{i+j} z$ and $u^j z$ is a prefix of v. First, $a \cdot u^i = a \neq b = b \cdot u^i$ holds, because of $a \cdot u = a, b \cdot u = b$ with $a \neq b$. On the other hand, $v = u^j zr$ for some $r \in \Sigma^*$, because $u^j z$ is a prefix of v. Therefore, using $a \cdot u^i = a, b \cdot u^i = b$, if $a \cdot u^{i+j} z = b \cdot u^{i+j} z$, then $a \cdot u^j z = b \cdot u^j z$ leading to

$a \cdot u^j zr = b \cdot u^j zr$ with $u^j zr = v$, which is a contradiction. Consider $w, z \in \Sigma^*$ with $u^\ell v = wz$ and $|z| = k$. We have already proved $a \cdot w \neq b \cdot w$. On the other hand, by our assumptions, $a \cdot wz \neq b \cdot wz$ and $\delta(a \cdot wzx) = \delta(b \cdot wzx)$. By Lemma 3.1, considering $a \cdot w, b \cdot w, a \cdot wz, b \cdot wz, z, x$ as c, d, a, b, u, x, we obtain that $\mathcal{A}$ is not k-reversible. Now, we assume that for every nonnegative integer k, the automaton $\mathcal{A}$ is not k-reversible. This means that $\mathcal{A}$ is not 0-reversible. Moreover, by Lemma 3.1, for every positive integer k, there are distinct $a, b \in A$, such that there exist $c, d \in A$, $u \in \Sigma^*$ with $|u| = k$, $c \cdot u = a, d \cdot u = b$, where we have either $a, b \in A_F$ or $\delta(a, x) \neq \delta(b, x)$ for some $x \in \Sigma$. Without any restriction we may assume that $k \geqslant |A|^2$. Obviously, by $a \neq b$, for every prefix w of u, $c \cdot w \neq d \cdot w$. But then, by $(k =)|u| > \frac{|A|(|A|-1)}{2}, u = x_1 \cdots x_k$ with $x_1, \ldots, x_k \in \Sigma$, there exists a repetition in the sequence $(c, d), (\delta(c, x_1), \delta(d, x_1)), (c \cdot x_1 x_2, d \cdot x_1 x_2), \ldots, (c \cdot x_1 \cdots x_k, d \cdot x_1 \cdots x_k)$ having $c \neq d$ and $c \cdot x_1 \cdots x_i \neq d \cdot x_1 \cdots x_i, i = 1, \ldots, k$. Thus, there are $p, r \in \Sigma^*, q \in \Sigma^+$ with $u = pqr$ and $c \cdot p = c \cdot pq, d \cdot p = d \cdot pq, c \cdot pq \neq d \cdot pq, c \cdot pqr \neq d \cdot pqr$ and either $c \cdot pqr, d \cdot pqr \in A_F$ or $c \cdot pqrx = d \cdot pqrx$ for some $x \in \Sigma$. By Lemma 3.1, this shows that $\mathcal{A}$ is not k-reversible. □

4. Conclusion

It is well-known that, by using characteristic sample sets, the canonical automata of k-reversible languages can be identified applying a time polynomial and space linear algorithm (this is a famous result of Angluin). In this paper the limitations of her method are shown. In other words, the characterisation is given for automata which are not k-reversible for any non-negative integer k. It is an interesting fact that this property was not investigated so far in the literature. Further work is to characterise classes of automata and their languages for which other learning algorithms can not be applied [2, 14, 7, 11].

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