

The evolvement of geometrical concepts in lower primary mathematics (Parallel and Perpendicular)

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Abstract

The evolvement of some geometrical concepts, such as parallel and perpendicular has been studied in a developmental teaching experiment in class four whose aim was to put the model of geometry teaching according to van Hiele into practice.

Our research question is how lower primary geometry teaching in Hungary, particularly teaching the concept of parallel and perpendicular is related to the levels formulated by van Hiele. Moreover to what extent are the concrete activities carried out at these levels effective in evolving the concepts of parallel and perpendicular.

Our hypothesis is that in lower primary geometry teaching (classes 1–4) only the first two stages of the van Hiele levels can be put into practice. By the completion of the lower primary classes transition to level 3 is not feasible. Although the set of concepts are evolved but there is not particular relationship between them. The logical relationships between the characteristics of a shape are not really recognized by children. They are not able to infer from one characteristic of a shape to another.

Keywords: mathematics teaching, parallel, perpendicular

MSC: 00A35, 97D70, 51F20

1. Introduction

Teaching geometry in Hungary in the first four grades of primary school aims at laying the ground to establish the skills through which learners can prepare for gaining knowledge on their own.

The basis of learning geometry is inductive cognition based on gaining knowledge. Starting out from the concrete and gathering experience from various activities will finally lead to the formulation of general relationships. The third educational principle laid down by Farkas Bolyai also emphasizes the importance of starting with the concrete: “(The teacher) . . . should always start with what learners can see and touch, and not with general definitions (it is not grammar that the first utterance is based on) and he should not torture prematurely with longwinded reasoning. . . We should start with geometric shapes and reading. . . and we also should get out of the sheet. . .” (Dávid, 1979)

In the framework curriculum what is particularly highlighted is the development of orientation in plane and shape, the formation of geometrical knowledge through recognizing geometrical shapes and the characteristics of forms and quantity as well as simple transformations.

In lower primary the basis of mathematical concepts is laid down. In this paper the development of the concepts of parallel and perpendicular is examined. In the framework curriculum the requirements of the teaching material related to parallel and perpendicular are as follows:

Grade 3: Measuring angles with non-standard units.

Grade 4: Producing plane figures by means of pairs of parallel and perpendicular straight lines. Measuring angles with right angle, its half and its quarter.

In some course books the concepts of parallel and perpendicular are introduced in various grades and in a different way. In a course book for grade 4. by C. Neményi it is during the presentation of the opposite and neighbouring faces of the rectangular solid and the cube that the concepts of parallel and rectangular are introduced and then they move on to the plane. In the course books by Hajdu, Török and Rakos these concepts are introduced in grade 3 when the position of pairs of straight lines in plane are studied. In the book entitled “Colourful mathematics” it happens in the same way, but only in grade 4.

The evolvment of some geometrical concepts such as parallel and perpendicular has been examined in an educational development experiment conducted with grade 4 pupils. The results and the lessons of the experiment are described below.

1.1. Linguistic problems

“One of the most important objectives of school is to provide children with a means i.e. language through which they will be able to learn, to think and speak about the world in which they actually live. Or to be more precise we intend to assist them to make this means they already possess more sophisticated.” (Holt, 1991)

Several children, like parrots, are able to repeat sentences containing technical terms, but they have not the slightest idea about their meaning. As a matter of fact this is what we would like to avoid. Some children on the other hand clearly understand the mathematical concepts and problems but they are at a loss when they should express their knowledge and thought in proper terms. Every one needs some time to incorporate words describing concepts into their active vocabulary and

they will be able to express their thoughts by means of proper terms. “Teachers can assist the process of incorporating words into learners’ active vocabulary in a way that they will not correct the improper language use all the time, moreover they try to understand children’s any, even inaccurate utterances in order that children could be able to become aware of their observations. But when the teachers want children to use a ‘technical term’ they have already understood, it is practical if they themselves repeat the utterance using the proper words.” (Szendrei, 2005)

According to Andre Revuz in every field of science, including mathematics, the most fearful obstacle to understanding is the language barrier.

What kind of context is accessible to fourth grade learners? Is the language of mathematics course books for lower primary learners suitable for the teaching material to be acquired?

Here is a selection of sentences related to parallel and perpendicular from mathematics course books for grade three and four:

“Parallel straight lines do not share any point, their position and distance is the same everywhere.” (Török, 2002)

“The length of perpendicular sections drawn between parallel straight lines is the distance between parallel straight lines.” (Hajdu, 2005)

“The pairs of straight lines in plane which do not have a point in common, no matter how much we make them longer, are called parallel straight lines.”

“Four angles made by two perpendicular straight lines are equal.” (Árvainé, 2005)

“The distance between parallel straight lines is everywhere the same.”

“Draw straight lines in a way that draw the lines along the two edges of the square ruler that make a right angle.” (Rakos, 2002)

In the framework curriculum the proper and exact level of mother tongue and technical terms suitable for the age group is emphasized. We wonder whether the above sentences are suitable for the language use of learners.

2. Theoretical background

2.1. The levels of geometrical thinking according to van Hiele

Young children start gaining knowledge in geometry already in kindergarten where the concept of geometrical objects (geometrical solids, plane figures) is being established by examining the shapes of the objects in the environment. Establishing the characteristics for the set of these objects implies a higher degree of gaining knowledge. A large amount of references can be found on gaining geometrical knowledge, but in this particular case we rely on van Hiele.

According to P-H van Hiele the process of gaining geometrical knowledge can be divided into five levels.

At the level of global cognition of shapes (level 1.) children perceive geometrical shapes as a whole. They easily recognize various shapes according to their forms, they remember the names of the shapes however they do not understand the

relationship between the shape and their components. They do not recognize the rectangular prism in the cube, rectangle in the square, because these are totally different things for them.

At the level of analysis of shapes (level two) children break shapes down into components and then put them together. They also recognize the faces, edges and vertices of geometrical solids as well as the plane figures of geometric solids which are delineated by curves, sections and dots. At this level particular importance is attached to observation, measurement, folding, sticking, drawing, modelling, parquetry, and using mirrors. By means of these concrete activities children can establish and enlist the characteristics of shapes such as the parallelism, perpendicular of faces and sides, characteristics of symmetry, the presence of right angle etc, but they are not able to define and to recognize the logical relationships between the characteristics. At this level children do not perceive the relationships between shapes.

At the level of local logical arrangement (level 3) learners are able to find relationships between the characteristics of a particular shape or between various shapes. They can also make conclusions from one characteristic of shapes to the other. They understand the importance of determination, definition. However the course of logical conclusions is set by the course book or the teacher. The need to prove things is started, but it applies only to shapes.

Level four (making efforts to reach complete logical set-up) and level five (axiomatic set-up) belong to the requirements of secondary and tertiary education.

In the van Hiele model each learning stage is constructed and enlarged by the thinking established by the previous stages. Transition from one level to the other happens continually and gradually, while children are acquiring the mathematical terms according to the particular levels. This process is particularly influenced by teaching, especially its content and method. For the suitable geometrical thinking none of the levels can be omitted. Every level has its own language, system of notation and logical set-up. From educational point of view it is highly relevant in the theory of van Hiele that we cannot expect from learners at a lower stage to be able to understand the instructions formulated in terms of a higher level. According to van Hiele this is probably the most frequent reasons for failures in mathematics teaching.

2.2. Concept formation

During the formation of a mathematical concept, the concept has to be fitted into the system of concepts established before (assimilation) but it can happen that the modification of the existing system or pattern is necessary for the fitting of the new concept. The balance of assimilation and accommodation is absolutely indispensable for the proper formation of concept. If this balance is upset i.e. assimilation is not followed by accommodation then the learners' own interpretations find their way into their mathematical knowledge, which later on may lead to misconceptions. Then the concepts formed in this way can be vague and inaccurate.

Teaching geometric concepts is as a matter of fact a long process. The principle

of progressiveness should be observed, and accurate definitions should be established but not by all means. Sometimes even at lower primary classes definitions are provided in course books, although learners lack the required experience and abstraction level. In this respect, what R. Skemp the mathematician and psychologist said is:

“By means of definitions it is impossible to transmit concepts to anyone which are at a higher level than his knowledge, only by providing plenty of proper examples. Since in mathematics these examples mentioned above are almost all of them various concepts, therefore we have to make sure that the learners have already acquired these concepts. Selecting the proper examples is a lot more difficult than we suppose. The example should possess those common characteristics which make up the concepts, but they should not have any other common characteristics.” (Skemp,1975)

The evolvement of scientific concepts, such as parallel and perpendicular is based on education. According to his observations Vigotsky came to the conclusion that “in as much as the progress of teaching contains the proper elements of the curriculum, the development of scientific concepts will proceed the development of spontaneous concepts.” In the progress of teaching the special co-operation of children and adults and the transmission of the teaching material in an order can give an account for the premature achievement of concepts. According to teaching experience it can be understood that the direct teaching of concepts is not really possible. The mere acquisition of a new word verbally covers only emptiness. In this case children acquire only words and not concepts. When children first recognize the meaning of a new word then the process of evolvement of a concept is being started. Scientific concepts are not acquired and learned “ready-made” by children but these concepts are evolved and established through the active thinking of children. The evolvement of spontaneous and scientific concepts is closely related to each other. A basic requirement for the evolvement and acquisition of scientific concepts is the proper level of spontaneous concepts. However the evolvement of scientific concepts can also have an influence on the development of spontaneous concepts.

Bruner’s representation theory is also based on activities: According to this theory in order that learners could understand the teaching material, they should “process” it intuitively before. According to Bruner every process of thinking can happen on three levels:

- a) enactive level: gaining knowledge through concrete practical activities and manipulations.
- b) iconic level: gaining knowledge through graphic images, imaginary situations.
- c) symbolic level: gaining knowledge through mathematical symbols and language.

Although in the lower primary grades the enactive and iconic levels are in the foreground, but language (speech), which is the symbolic level, is also very important. If we get round the first two levels there is a risk that learners due to the lack of proper system of images will not be able to solve mathematical problems and to

understand concepts at symbolic level, because they have nothing to rely on. If the first level (the concrete, practical activities) is omitted, then the proper system of images will not be established.

“The concept image is the total cognitive structure associated with the concept name, which includes all the mental pictures and associated properties and processes, pictures, graphs experiences”. (Tall, 2004)

In our teaching experiment learners have gained a wide range of experience of the concepts of parallel and perpendicular through various concrete activities such as modelling, folding, clipping and drawing. Thus, their concept image will be versatile.

3. A developmental teaching experiment

3.1. Research question

The research question raised is what the relationship is between lower primary geometry teaching including the teaching the concept of parallel and perpendicular and the geometric levels according to van Hiele. Furthermore how efficiently the concrete activities at these levels contribute to the establishment of the concept of parallel and perpendicular.

3.2. Hypotheses

In lower primary (grades 1–4) geometry teaching can reach the first two stages of geometric thinking according to the van Hiele levels. It is not feasible to reach level 3 by the completion of lower primary. Although sets of concepts are established, but there is no relationship whatsoever between them. Actually children do not recognize the logical relationships between the characteristics of a shape and they are not able to draw conclusions from one characteristics of a shape to another.

3.3. Research methodology

It was in May–June 2006 that the teaching experiment was carried out whose content and method was devised by the author, who also was involved in the lessons. The teacher was a mathematics teacher and supervisor in class 4.c of the Practice School of József Eötvös College in Baja, whose job was assisted by the author in presenting, modeling and eventually raising supplementary questions or alternative explanations. Both of them helped the pupils in carrying out and solving the tasks in individual or pair work. Both checking and evaluation were done in cooperation. During the developmental teaching the evolution of several geometrical concepts such as rectangles, squares, parallel, perpendicular and symmetry was examined but here we are going to present the formation of the concept of parallel and perpendicular.

The developmental teaching experiment included 16 lessons and the aim was to put the van Hiele model of geometry into practice. In the first lesson a pre-test was done by 26 pupils of class 4 so that we could see that the transition from level 1 (the global recognition of shapes) to level 2 (the analysis of shapes) and the further development of geometric thinking is feasible. When compiling the pre-test, the syllabus of class 3 and the comments of the mathematics teacher were taken into consideration. The first lesson of the development teaching experiment was also the first lesson of the geometry topic as well.

3.4. Pre-test

The task of pre-test 3 was to reveal the conceptual level of the right angle and the angle smaller and bigger than right angle. The task for the children was to decide about angles of seven plane figures as to which of them are right angles, smaller or bigger than right angle and they coloured the angles red, blue and green respectively.

The angles of the plane figures below were examined by the children:

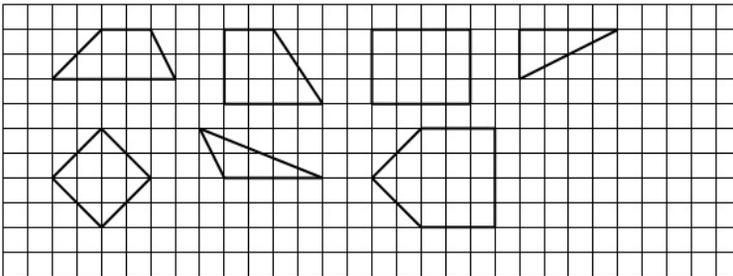


Figure 1: Pre-test

The results are shown in the following table:

The size of every angle is correct.	30.8%
Every right angle was marked properly.	46.2%
One mistake in finding right angles.	15.4%
Right angles were marked properly only in squares and rectangles.	26.9%
Every acute angle and obtuse angle was properly coloured.	30.8%
Two or more mistakes were made in finding acute angles and obtuse angles.	69.2%

As it is revealed by the data the concept of angle needs further development. Almost 70% of the learners made mistake in establishing the size of angles. They proved to be most successful in designating right angles, almost half the group came up with the right solutions. However we still cannot be satisfied with this result.

Tasks related to parallel and perpendicular were not given since children were not familiar with these concepts.

3.5. A developmental teaching experiment

When compiling the teaching material the principle of gradualness was observed and the problems were made more and more difficult. During the first lessons we focused on the characteristics of rectangular solids and cubes. When examining the position of opposite and neighbouring faces the new concept of parallel and perpendicular were introduced. In case of various solids the position of the opposite and neighbouring faces were examined then after spreading the solids we moved on to the plane. In the plane first learners came across with parallel and perpendicular when studying the opposite and neighbouring sides of rectangular and squares.

The detailed description of the lessons can be found in the supplement.

When designing the lessons what we considered of utmost importance was that children could discover geometrical concepts first through concrete experience in real games and activities, later at visual level (drawing) then at an abstract level.

3.5.1. Concrete, manual activities

- a) Showing parallel/non-parallel using both hands in various positions.
 - b) Showing perpendicular/non-perpendicular using both hands in various positions.
 - c) Producing perpendicular/non-perpendicular position with the leaves of the course book.
 - d) Finding the opposite and neighbouring faces of the regular pentagonal prisms and square based pyramids, studying their position from the point of view of parallel and perpendicular.
 - e) Producing plane figures from two coincident right angled triangles. Studying the parallel and perpendicular opposite and neighbouring sides of the quadrangles produced in this way.
 - f) Demonstrating parallel and perpendicular pairs of straight lines in plane by means of two skewers, also demonstrating non-parallel and non-perpendicular straight lines.
- Etc.

Minutes were taken of every lesson, in which the children's responses were also recorded. When examining the position of the faces of the regular pentagonal prism the following conversation took place:

-Show me please which face is opposite this lateral face? What do you think, Petra?

-This one and that one. (She pointed at the two non-neighbouring faces.)

-Are the opposite faces parallel or not?

-No, they aren't.

-Why not?

-Because they are slanting...

-Are these two neighbouring lateral faces parallel or not? (The teacher pointed at them.)

- They are not parallel because they meet.
- Are they perpendicular to each other?
- No, they aren't.
- Why not?
- Because they do not make a right angle, I have checked it with a folded right angle.

In task f) when demonstrating the parallel position, after considering several solutions, Szabolcs came up with the following statement: "It did not matter either how far the skewers were from each other."

Beside the examples demonstrating the concept, examining counter examples is also essential in order to establish a clear-cut concept. After analyzing several examples and counter examples children will reach the level, where they will be able to recognize the essential characteristics of a concept and they will be able to differentiate between the essential and the irrelevant characteristics.

3.5.2. Visual tasks

- a) Drawing parallel and perpendicular pairs of straight lines on grid.
 - b) Drawing various triangles on grid and colouring the sides perpendicular to each other.
 - c) Colouring the parallel lateral pairs of various plane figures, designating right angles.
 - d) Drawing quadrangles of given characteristics.
 - e) Sorting out plane figures according to given characteristics.
- Etc.

In task c) when colouring the parallel sides of a general trapezoid, the following conversation took place between the author and the child called David:

"You haven't coloured anything in this quadrangle. Haven't you found parallel straight lines?"

"No, I haven't."

At this point the teacher in order to help the child placed the skewers on the two bases, thus David could see that they do not meet. David made the following remark:

"It is true that the skewers do not meet, but the sides are not of equal length, thus they cannot be parallel."

"But this was not a condition for parallel."

Then David said:

"Well, in this case, they are parallel."

Then he corrected the mistakes, which may have come from the fact that most of the time was devoted to the characteristics of square and rectangle. In these quadrangles beside the fact the opposite sides were parallel, they were also of equal length. David probably connected these two characteristics.

3.5.3. Abstract level

After gaining experience at the previous two levels the characteristics of various geometrical shapes were summarized at an abstract level:

- in case of solids, especially cubes and rectangle prisms counting the number of faces, edges and vertices repeatedly, determining the length of edges, the parallel and perpendicularity of faces and edges, and the number of symmetry planes;

- in case of polygons, especially squares and rectangles counting the sides and vertices repeatedly, examining the length and parallel and perpendicularity, determining the number of symmetry axes, and the size of angles produced by the neighbouring sides. Obviously the geometric characteristics were studied through models or the visual representation of the given shape.

Twenty questions is one of the favourite games among children, which is also suitable for practising the characteristics of solids and plane figures. During a game what the children had to guess was the rectangle. These are the questions of a game:

- Does it have five vertices?
- Is it a quadrangle?
- Does it have a right angle?
- Are the opposite sides parallel?
- Does it have a symmetry axis?
- Does it have two symmetry axes?
- Are the sides of the same length?
- Does it have perpendicular sides?

At this point the teacher said they could have guessed it from an earlier question.

- Does it have several right angles?
- Does it have four right angles?

Finally they found out what it was.

During the game of twenty questions we wanted the children to realize that instead of just guessing it is a good strategy to limit the options. We had to convince them that they should not be afraid of asking questions and they should also see that some questions are more purposeful than others and “no” to a good question is just as good as a “yes”. Moreover it is no use asking a question when they already know the answer.

3.6. Post-test

The developmental teaching experience was completed by an evaluation worksheet, which was filled in by 25 learners in class 4.c, 23 learners in class 4.a and 24 learners in class 4.b respectively. In these latter two classes the mathematics teacher - supervisor was another teacher.

In the worksheet it was only the questions related to the establishment of the concepts of parallel and perpendicular that were evaluated.

In the task of recognizing the parallel and perpendicular lateral pairs of faces of polygons, children were asked to examine nine shapes. They also did the following

tasks:

- a) colouring the parallel sides using the same colour;
- b) colouring the right angles red;
- c) enlisting the letters designating the plane figures which have parallel sides;
- d) enlisting the letters designating the plane figures which have perpendicular sides.

sides.

The following polygons have been examined:

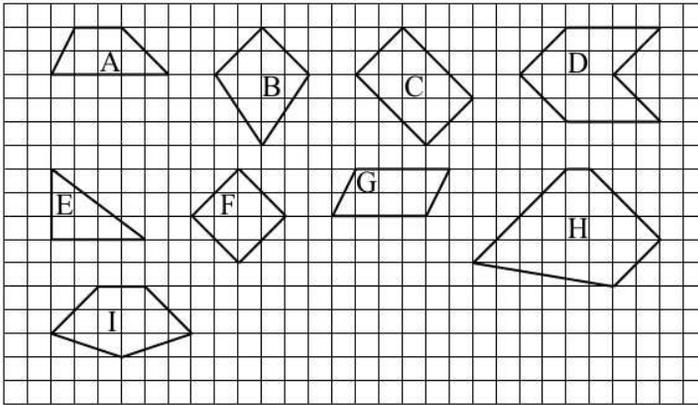


Figure 2: Post-test

The results of the tasks are shown in the chart below:

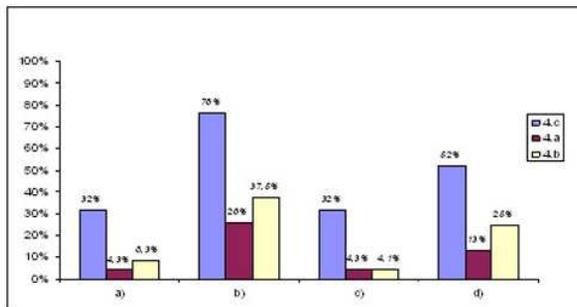


Figure 3: Correct solutions

In the experimental group children’s best results were gained in colouring right angles. It was in grade 3 that they came across this concept and it was further developed in grade 4. The outcome of the experiment is shown by the fact that the number of learners who correctly marked the right angle increased from 46% in the pre-test to 76% in the post-test. The results in the control group were significantly

worse, 26% and 37% respectively. The difference between experimental and the control group was also considerable in the recognition of parallel and perpendicular lateral pairs.

In the task of the worksheet related to the characteristics of squares and rectangle children were asked to underline the statements which were true for

a) squares:

- Its opposite sides are parallel.*
- Its opposite sides are perpendicular.*
- The neighboring sides are parallel.*
- The neighboring sides are perpendicular.*
- Every angle is right angle.*
- Not every angle is right angles.*
- It has exactly two symmetry axes.*
- It has four symmetry axes.*
- It has 8 symmetry axes.*
- Its every side has same length.*
- Its opposite sides have same length.*

b) rectangles:

- Its opposite sides are parallel.*
- Its opposite sides are perpendicular.*
- Its neighboring sides are parallel.*
- Its neighboring sides are perpendicular.*
- Its angles are all right angles.*
- Its angles are not all right angles.*
- It has four symmetry axes.*
- It has four symmetry axes.*
- The diagonals are symmetry axes.*
- Every side has same length.*
- The opposite sides have same length.*

In the evaluation of the tasks we have focused on only the results without any mistake. In the experimental group all the true statements related to the properties of the square were chosen by 52 of the pupils, whereas in the control group the results were 35% and 42%. In case of rectangle the results were as follows: In the experimental group it was 64% in the control groups 39% and 70%. The mistakes can be due to the misunderstanding of the terms 'opposite' and 'neighboring' on the one hand and in the fact the concepts of parallel and perpendicular were not firmly established.

During the developmental experiment only the initial steps were taken to establish the concepts of parallel and perpendicular.

4. Conclusion

The developing teaching experiment guided by the author efficiently contributed to the establishment of the concept of parallel and perpendicular, and the comparison of the results of the pre-test and the post-test also supported the above finding. In the experimental group the results were significantly better than in the control groups. Our findings are related to only the samples examined, which are not representative, and therefore no statistical trials have been carried out. The data measured, the interviews and the games support the hypothesis that it is not possible to reach level 3 of geometrical thinking according to van Hiele by the completion of lower primary (the first four classes of primary education), only reaching the first two levels is feasible. Children are not really able to make conclusions from one characteristic of the figures to the others. They cannot find the relationships between the characteristics of a given figure.

The cognition of children of 6–10 years olds is highly attached to real life, which is why during the formation of concepts only starting out from concrete, manual activities and examples taken from their immediate experience is it possible to reach the level of abstraction. A large number of examples and counter example and making the concept concrete several times and modelling are the preconditions that make it possible for children to recognize the essential characteristics of a concept and they could reach the level of abstraction.

As György Pólya said: “We should not pass up anything that could bring mathematics closer to students. Mathematics is a very abstract science and this is why it has to be presented in a very concrete way.” (Pólya, 2000)

5. Supplement

Lesson 1: Pre-test.

Lesson 2: Naming and describing rectangular objects, such as matchboxes, cupboards etc, the number of vertexes, edges and sides, comparing the length of edges, the shape and the size of the sides, understanding what opposite and neighboring sides are and their position. Naming and describing cubic objects: the number of vertexes, edges and sides, comparing the length of the edges, the shape and size of sides, understanding what opposite and neighboring sides are and their position.

Lesson 3: Giving a list of the characteristics of rectangular prisms and cubes by means of models. Making up various rectangular prisms using four matchboxes. Producing the reflections of the solid made from matchboxes. Finding objects in symmetrical arrangement in the classroom. Listing symmetrical objects. Defining the position of the planes of symmetry in case of various solids.

Lesson 4: Defining the position of the planes of symmetry in rectangular prisms and cubes. By using a model, learners studied the parallel and perpendicular

position of the opposite and the neighboring sides of rectangular solids, regular pentagon prisms, quadrilateral pyramids. Spreading rectangular prisms and cubes, examining the shape and size of the sides. Cutting squares from rectangles.

Lesson 5: The various grids of cubes. Studying the rectangles. The number of vertexes, opposite and neighboring vertexes, diagonal. Cutting the rectangle into two along the diagonal. Producing other plane figures by fitting the triangles gained in this way, and naming them. Gathering experience on plane figures and describing them. Further study of the rectangles: the number of sides, comparison of their length, determining the opposite and neighboring sides, the parallel position of the opposite sides, the perpendicular position of the neighboring sides. Measuring the sizes of angles by means of folded right angles.

Lesson 6: Studying the characteristics of plane figures made from two congruent right-angled triangles during the previous lesson: the number of vertexes and sides, defining the opposite and neighboring vertexes and sides, comparing the length of the sides, studying the parallel and perpendicular position of the opposite and the neighboring sides, the size of the angles. Comparing the characteristics of rectangles and parallelograms and highlighting their differences. Studying squares: the number of vertexes, opposite and neighboring vertexes, the diagonal. Cutting the square into two parts along the diagonal. Producing plane figures from the two right-angled isosceles triangles. Further study of squares: the number of sides, comparing their length, opposite and neighboring sides, the parallel and perpendicular position of the opposite and neighboring sides, the size of the angles.

Lesson 7: Demonstrating parallel and perpendicular pairs of straight lines as well as straight lines which are not parallel and perpendicular. Producing plane figures cut out from paper without restriction, and describing their characteristics. Listing the characteristics of rectangles and squares. Producing plane figures from the 2, 3, 4 and 6 regular triangles from the set of logics, which consists of 48 various plane figures, which can be red, yellow, blue or green. Their sizes are, small or large, their shape can be circle, square or triangle, their surface can be smooth or there is a hole in them. Making observations on parallel pairs of sides. Producing rectangles of different length and identical height from strips of paper.

Lesson 8: Producing various plane figures from paper strips by one cut. Naming them and describing their characteristics and shared characteristics. Cutting general rhombus from rectangle, its characteristics. Cutting general deltoid from rectangle, and its characteristics. Making rectangles and then the “frame” of a general parallelogram from six match sticks. Making squares then general rhombus from four match sticks. Comparing the characteristics of squares and rhombuses.

Lesson 9: Comparing the characteristics of squares and rectangles. Making 2 rectangles, a pentagon and a triangle, a triangle and a quadrangle, 2 quadrangles and 2 triangles from a rectangle by one cut. Drawing squares on square grid.

Lesson 10: Drawing various quadrangles on square grid. Drawing various triangles on grid. Drawing parallel and perpendicular pairs of straight lines.

Lesson 11: Coloring the parallel pairs of sides of the quadrangles drawn on grid and designating the right angles. In triangles coloring the sides perpendicular to each other. Drawing quadrangles according to given requirements. Studying the structure of the edges of rectangular prism and cubes. Observing the parallel and perpendicular edges.

Lesson 12: Producing reflection on plane through activity: folding a painted sheet of paper, on a black photographic paper folded into two making a pattern by running a pin through it, then unfolding it holding it in the direction of light. Cutting a given pattern from a sheet of paper folded into two parts. Observing reflections. On grid reflecting given figures on given axis. Producing figures symmetrical on axis by clipping.

Lesson 13: Finding the symmetry axes of plane figures cut out from paper by means of folding and mirror. Formulating experiences and observations. Drawing plane figures which have no symmetry axis, and which have exactly 1, 2, 3 and 4 symmetry axes.

Lesson 14: Producing figures symmetrical on axis on square grid. Selecting plane figures according to given characteristics. Formulating statements “every” and “there is such...”.

Lesson 15: Selecting plane figures according to given characteristics. Establishing the logical validity of statements. Drawing plane figures according to given requirements. Twenty questions.

Lesson 16: Post test.

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