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# A special localization algorithm in Wireless sensor networks for telemetry application

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**Abstract**

Accurate positioning [1] in Wireless sensor networks [3] is an important and emerging technology for many research areas, such as health care, telemedicine [2], sports, commercial, public-safety and military applications. This paper presents an exact positioning approach for a telemetry system in ad hoc sensor network measuring patients' vital and motion parameters. The telemetry system includes two kinds of wireless data-gathering devices (sensors) neither of which knowing their own positions: 1. reference point devices, which retain fix positions, and are uniformly distributed over an area of interest, 2. all the other sensors (mobile nodes) which are allowed to change their positions during the measurements. The presented algorithm is based on measurements of distances between sensor nodes with 4 Hz sampling frequency. The aim is to compute the exact positions of the nodes in some fixed coordinate system. The measurements of distances between sensor nodes are not sufficiently accurate. The developed algorithm at first estimates the exact position of the reference points from initial measuremets based on the minimization of the localization error. During the measurements the positions of the reference points do not change. The mobile nodes localize themselves continuously with the help of location references received from the reference points using trilateration.

*Keywords:* Wireless Sensor Networks, Localization algorithms, Telemetry system

*MSC:* AMS classification numbers

# 1. Introduction

A wireless sensor network [3] consist of spatially distributed autonomous sensor nodes for data acquisition. Each node is able to sense the environment, perform simple computations and communicate with its other sensors or with the central unit.

Accurate localization of sensor nodes is a strong requirement in a wide area of applications such as health care, telemedicine, sports, rescue, disaster relief, commercial and military and applications [2].

This paper describes an algorithm for localization of sensor nodes using range measurements, trilateration and heuristic geometrical approach.

The paper is organized as follows. Section 2 presents the basic problem, the formulation of localization problem in wireless sensor networks, and the applied exact positioning approach for the I-QRS Telemetry system. In section 3 the developed localization algorithm is presented. In section 4 the applied heuristic and the implementation results are described. Future and Related work has been discussed in section 5 and 6, respectively.

# 2. The basic problem

## 2.1. Wireless sensor networks (WSN)

A wireless sensor network (WSN) [3] has important applications such as remote environmental monitoring, health monitoring and target tracking. The type of wireless sensor network investigated here refers to a group of sensors, or nodes, that are equipped with wireless interfaces through which they can communicate with one another to form a network. Each node is able to sense the environment, monitor physical or environmental conditions (such as temperature, sound, pressure, etc). The sensors perform simple computations, and cooperatively pass their data through the network to a main location. The design of a WSN depends significantly on the application, and can vary from a simple star network to an advanced multi-hop wireless mesh network.

## 2.2. Localization algorithms

The theoretical background of localization is presented briefly in this section. Please find more details on the topic in references [1, 7]. The goal of localization is to determine the physical coordinates of a group of sensor nodes. Many methods have been proposed in the literature and used in practice to localize wireless devices.

Localization algorithms can be classified into *exact* and *approximative* localization.

*Exact* localization is based on precise measurements of distances or angles between sensor nodes not knowing their own position and nodes with preinstalled localization systems. These methods result high precision of position determination but need extensive calculations.

*Approximative* algorithms do not require extensive calculations and result in less network traffic. From network managemenet aspect the research can be classified into two categories: *centralized* and *distributed* localization.

In the *centralized* localization it is not necessary to compute each node. Sensor nodes gather environmental data and pass it to a base station. After analysis the computed positions are transported back into the network. Disadvantage of this method is high communication costs.

In the case of *distributed* localization algorithms all computations are done on the nodes. The sensor nodes communicate with each other to get their positions in a network.

In the network model, the nodes are located in distinct physical locations in some region of space. We assume below that nodes have some means by which they can measure the distance between themselves.

A wireless ad-hoc network can be modeled by a distance graph $G = (V, E, D)$, where $V$ is the set of wireless nodes, $E$ is the set of links, and $D(u, v)$ denotes the distance measurements between a pair of nodes $u$ and $v$. An important question is whether or not a network is localizable by given its distance graph.

Let $N$ be a network in $R^2$ with nodes labeled $0, 1, ..., n$, and assume $G$ has a bilateration ordering. A sensor network with a total number of $n$ nodes consists of $k$ sensor nodes and $b$ beacons (anchors) ($b << k$), where **beacons** are able to determine their own position, and the positions of **sensor** nodes are not known. Note that the distance between any two anchors is known since the positions of all of the anchors are known. The positioning error of these localization systems depends on the quality of the used localization devices.

## 2.3. The applied exact positioning approach

This paper presents an *exact centralized positioning approach.* The presented algorithm is based on measurements of distances between sensor nodes with 4 Hz sampling frequency. None of the nodes knows their own position, so there are not beacons, but there are two kinds of sensor nodes: **reference points** (fix) and **mobile nodes** (changing its position continuously). The **reference points** do not know their own position, and the measurements of distances between sensor nodes are not sufficiently accurate, but during the measurement their positions are fixed. The first step is to compute the exact positions of the reference nodes in some fixed coordinate system. After getting the positions of the reference points, they are fixed during the measurements, and the **mobile** nodes localize themselves continuously with the help of distance measurements received from the reference points. The aim is to compute the exact positions of the reference points and mobile nodes in some fixed coordinate system.

### 2.4. The I-QRS Telemetry System

The IQRS Sport Telemetry System [10] monitors the vital and motion signs of the player continuously. It records and transmits data in real-time by wearing a chest belt. The local computer receives the signals, uses automatic data-analysis and alarm algorithms, and transfers the signals through a data channel (mobile phone, broadband internet, or other similar connections) to an internet connected server, where an expert system further processes the signals. The trainers and doctors may use a web-based system to access the data. *The motion analysis and animation module of the system is based on the positioning algorithm presented in this paper.* With the help of the animation module the movement of players and their techniques can be monitored and also played back in 3D format after the trainings. In the system the number of reference points is smaller then 10, and the number of mobile nodes is bellow 100, and the size of the sport field is about 250 meter.

## 3. The developed exact positioning algorithm

The developed positioning algorithm is optimized for the I-QRS Telemetry system. The aim was to develop a sufficient algorithm for each step of the telemetry system's process, starting from the setting up of the device, during the whole measurements until the analysis ends. Our aim was not to develop a new optimal algorithm for large networks, but to analyze the existing techniques, combine and modify them, and develop a new heuristic according to the needs of the telemetry system for daily use. The developed algorithm consists of two main steps.

**Algorithm I. : Localisation of the reference points**

The developed algorithm at first estimates the exact position of the reference points from initial measurements based on the minimization of the localization error. During the measurements the positions of the reference points do not change.

**Algorithm II. : Localisation of the mobile nodes**

The mobile nodes localize themselves continuously with the help of location references received from the reference points using trilateration.

### 3.1. Algorithm I.: Localisation of the reference points

This iterated algorithm can be used to provide the locations of the reference points. In each iteration an initial set of two nodes is fixed and used to define a coordinate system, and each node uses distance estimates to each other to solve a set of circle-circle intersection problems, solved through a coordinate geometric formulation. The computed possible positions of the reference nodes form polygons.

The resulting polygons of the iterations are transformed to a common coordinate system, where the exact positions of the nodes are computed with respect to the minimization of the localisation error. The examples are presented for 8 reference point, since the I-QRS Telemetry system uses 8 reference nodes.

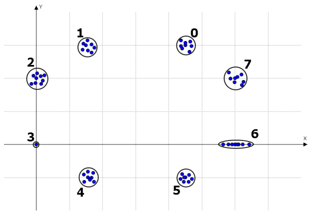


Figure 1: Octagons

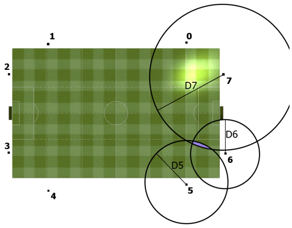


Figure 2: Trilateration

**Input:** *Measurements of distances between reference nodes* $(Ref_1, \ldots, Ref_n)$ *with 4 Hz sampling frequency*

**Output:** *The exact positions of the reference points in a fixed coordinate system*

#### 3.1.1. Step 1. Initialization

- **collect** a few thousand distance measurements:

$$distances[Ref_i, Ref_j](0 \leq i, j \leq n)$$

- **for** all pair of the reference points $(Ref_i, Ref_j)$ **do average** the values, and **filter** the outliers
- the **output** is a distance matrix $D$, where $D(Ref_i, Ref_j)$ denotes the distance between reference points $Ref_i$ and $Ref_j$

#### 3.1.2. Step 2. Computing the polygons

**for** all pair of the reference points $(Ref_i, Ref_j)$ **do**

- Estimate the coordinates of the other nodes ($Ref_k : 0 \leq k \leq n, k \neq i, j$) using coordinate geometry of circles, solving the circle-circle intersection problem
- Two circles may intersect in two imaginary points (throw away), a single degenerate point (good), or two distinct points ($k1(x1, y1)$ and $k2(x2, y2)$- the choice is based on the minimization of the localization error )

$$u = d(i,k)^2 - d(j,k)^2 + x_i^2 + x_j^2 - 2x_ix_j - y_i^2 + y_j^2; v = x_j - x_i; w = y_i - y_j$$

$$k_{y1/2} = \frac{-4(uw - 2v^2y_i) \pm \sqrt{4(uw - 2v^2y_i)^2))}}{2 * 4(w^2 + v^2)} -$$

$$\frac{-4(uw - 2v^2y_i) \pm \sqrt{-4 * 4(w^2 + v^2)(u^2 - 4v^2(d(i,k) - y_i^2))}}{2 * 4(w^2 + v^2)}$$

$$k_{x1/2} = \frac{x_j^2 - x_i^2 - ((k_{y1/2} - y_i)^2 - d(i,k)^2) + ((k_{y1/2} - y_i)^2 - d(j,k)^2)}{2v}$$

For every iteration the result is a polygon. The number of iteration step is $n * (n - 1)/2$.

#### 3.1.3. Step 3. Transformation of the polygons in a common coordinate system

The identification number of the reference points, and the polygons angels and side lengths are known, so they can be transformed to a common coordinate system ($Ref_i$ is in the **origo** and the coordinate of $Ref_j$ is $(D(Ref_i, Ref_j), 0)$) for some fixed $i$ and $j$. The output is a set of polygons, and a set of computed possible coordinate for each reference point (Figure 1).

#### 3.1.4. Step 4. Find the exact positions

The aim is to find the exact position for every reference point. The goal is to minimize the sum of squares of the errors between the real positions of the references and their computed possible positions with respect to the measured distances ($D(Ref_i, Ref_j)$) and distances computed from the possible positions ($\bar{D}(Ref_i, Ref_j)$).

$$min \sum_{i,j,i<j} (\bar{D}(Ref_i, Ref_j) - D(Ref_i, Ref_j))$$

Since the search space is too large we can calculate the average of the possible positions or apply beam search or other searching techniques to solve this problem. In the implementation we have used a special heuristic described in Section 4.

## 3.2. Algorithm II.: Localisation of the mobile nodes

The mobile nodes localise themselves continuously with the help of distance measurements received from the reference points using trilateration (Figure 2.). The trilateration algorithm is applied for every set of three reference nodes in Step 1, the result of this step is a set of possible mobile node positions. In Step 2 the optimal positon is computed based on the set of possible mobile nodes positions. The **input** of the algorithm is set of distance measurements between the mobile node and reference points, and the **output** is the exact position of the mobile node.

### 3.2.1. Step 1. Trilateration for every set of three reference nodes

Denote $P(x,y)$ the coordinate of the mobile node have to be positioned, , $r_\sharp$ the measured distances from the three actual reference nodes indexed by $i_\sharp$.

**for** each mobile nodes **do for** each set of three reference nodes **do**

- The trilateriation part (Figure 2): format the matrixes based on Pythagoras theorem

$$\begin{bmatrix} (x-x_{i1})^2+(y-y_{i1})^2 \\ (x-x_{i2})^2+(y-y_{i2})^2 \\ (x-x_{i3})^2+(y-y_{i3})^2 \end{bmatrix} = \begin{bmatrix} r_{i1}^2 \\ r_{i2}^2 \\ r_{i3}^2 \end{bmatrix}$$

- The least square algorithm: derive the matrix above to get the format: $(H * P(x,y) = Z)$. To do this subtract the first equations from the others. After some algebric manipulation we get the following:

$$\begin{bmatrix} 2x_1-2x_2 & 2y_1-2y_2 \\ 2x_1-2x_3 & 2y_1-2y_3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} r_2^2-r_1^2+x_1^2-x_2^2+y_1^2-x_2^2 \\ r_3^2-r_1^2+x_1^2-x_3^2+y_1^2-y_3^2 \end{bmatrix}$$

This is a least square system $(H * P(x,y) = Z)$ which has the following solution for $x$ and $y$

$$H^T * H * P = H^T * Z$$

$$(H^T * H)^{-1} * (H^T * H) * P = (H^T * H)^{-1} * H^T * Z$$

$$P = (H^T * H)^{-1} * H^T * Z$$

The output of this step is for each mobile node a set of possible position $P(x,y)$.

### 3.2.2. Step 2. Choose the optimal position

This step compute the optimal position for each mobile node from a set of computed positions of the mobile nodes.

1. filter the to the outliers from the set of computed positions $P(x,y)$
2. **for** each set of three reference nodes **do** compute the following
   - $err_i = |r_i^2 - ((x-x_i)^2 + (y-y_i)^2)|(i = 1,2,3)$

- $err = err_1 + err_2 + err_3$

3. The position having the minimal value of err is the **output**

### 3.3. Analysis and Optimization of the algorithm

The algorithm described above is a general solution. The exact identification of the class of network types that can be completely localized using this algorithm is a task of the future work. In the implementation a special heuristic is used during the computation of the polygons in order to reduce the integration errors. The implementation results show that the localizability depends on the quality of the distance measurements and on the shapes of the polygons formed by the reference points. Different optimization approaches can be applied to improve the accuracy of the positions.

- Iterated trilateration [4] can be used for three reference points instead of solving the circle-circle intersection problem for two reference nodes in each iteration
- The polygons can be weighted according to an error function
- Error model can be used which depends on the quality of the used localization devices (Temperature, Power, Clock skew, Moisture, Reflection, Constant error)

## 4. Implementation results

The localization algorithm is optimized for the I-QRS Telemetry system [10], in which the number of the reference points is smaller then 10, and the number of the mobile nodes is bellow 100. In the localisation of the reference nodes (Algorithm I.) a special heuristic is used. The polygons are created in a special order iteratively and they are weighted according to the iteration number. For all reference point triplets a triangle is created, and the triangle with the largest area is choosed in the first iteration to create the polygon (solving the appropiate circle-circle intersection problems). In the second iteration step in similar way the triangles having common side with the first triangle are chosen to create the polygons. This method is done for three iteration (Figure 4). After transforming and rotating the polygons in a common coordinate system, the final positions of the reference points are calculated with respect to the weights computed in the different iteration steps. We first evaluated the performance of the algorithms with generated precise distance measurements. The result was very good, the accuracy is lower than 0,001 meter.

The distance measurements of the I-QRS telemetry system in everyday use are often very noisy (the average accuracy is 3 meter), and on many sport fields the reflexion (the measured distance is about double of the real distance) is very high.

The test results of the algorithm show that the accuracy of the reference points positioning algorithm is about 0,3 meter, and of the mobiles nodes calculation algorithm is about 1,2 meter. To improve further the quality of the system, the calculated positions are filtered using different kinds of filtering methods.

# 5. Future work

An exact centralized positioning approach was presented in order to determine positions on the basis of distance measurements. An important goal of the further work is to improve the accuracy of the positioning algorithm. Two promising approaches are the application of quaternion based algorithm presented in [5], and using Klaman filters [6] based on the information of other sensors (gyroscope, accelemrometer, magnetometer). Figure 3 shows the work under development.

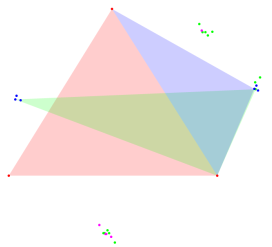

Figure 3: Three iteration

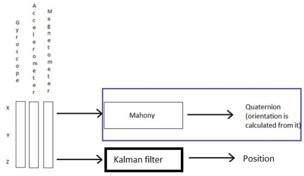


Figure 4: Future work

# 6. Related Work

Localization in WSN is an active area of research and so there are some existing literature surveys [1,7] on this topic. A promising class of approach for precise localization is fine-grained localization [8].

This paper presents an exact centralized positioning approach. The idea of the presented algorithm is related to simple iterated trilateration and the so called Sweeps algorithm [9]. In iterated trilateration, an initial set of three nodes is fixed and used to define a coordinate system. At each stage of the algorithm, there is a set of localized nodes and a set of unlocalized nodes. If an unlocalized node has distance measurements to at least three localized nodes, its position will be alculated and it will be added to the set of localized nodes. Simple iterated trilateration is suboptimal in that there are many localizable networks which it cannot localize. In [9] a class of algorithms is described for fine-grained localization called Sweeps. Sweeps correctly finitely localizes all nodes in bilateration networks. Sweeps also handles angle measurements and noisy measurements. The algorithm presented in this paper is based range measurements, trilateration and heuristic geometrical approach. Some step of the developed algorithm shows similarities to the mentioned approaches above, but our algorithm covers each step of the telemetry system's process, starting from the setting up of the device, during the whole measurements until the analysis ends. We developed a new heuristic optimized to the needs of the telemetry system for daily use.

# References

[1] Amitangshu, P., Localization Algorithms in Wireless Sensor Networks: Current Approaches and Future Challenges, *Network Protocols and Algorithms*, Vol. 2 (2010), No. 1., ISSN 1943–3581.

[2] Rashid, L. B., On the Definition and Evaluation of Telemedicine, *Telemedicine Journal*, Vol. 1(1) (1995), 19–30.

[3] Jennifer, Y., Biswanath, M., Dipak G., Wireless sensor network survey, *Computer Networks*, Vol. 52 (2008), Issue 12, 2292-–2330.

[4] Zheng, Y., Yunhao L., Confidence-Based Iterative Localization. Parallel Distrib. Syst., IEEE Transactions 21(5) (2010), 631–640.

[5] MAHONY, R., HAMEL, T., Pflimlin, Jean-Michel, Nonlinear Complementary Filters on the Special Orthogonal Group, Automatic Control, IEEE Transactions, Vol. 53 (2008), Issue 5, 1203-–1218.

[6] Chen H., A robust location algorithm with biased extended Kalman filtering of TDOA data for wireless sensor networks, *International Conference on Wireless Communications, Networking and Mobile Computing, Proceedings*, Vol. 2 (2005), 883—886.

[7] J., Bachrach, C., Taylor, Localization in Sensor Networks, *Handbook of Sensor Networks: Algorithms and Architectures*, (I. Stojmenovic, Ed.), (2005)

[8] J., Albowicz, A., Chen, L., Zhang, Recursive position estimation in sensor networks. *In Proceedings of the 9th International Conference on Network Protocols* (2001.), 35-–41.

[9] David, K., Goldenberg et. all, Localization in Sparse Networks using Sweeps, MobiCom '06, *Proceedings of the 12th annual international conference on Mobile Computing and Networking*, 110–121.

[10] `http://www.iqrs.hu/`