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COMPOSITE SPACETIME FROM TWISTORS AND ITS EXTENSIONS

ABSTRACT: *The main ideas of the twistor and supertwistor descriptions of spacetime and superspace in* D=4 *and* D=6 *dimensions are considered briefly from a didactical point of view. We underline also the role of complex twistor formalism for* D=4 *and the quaternionic twistor description for* D=6 *dimensions.*

1. Introduction.

The theory of twistors has been formulated by Roger Penrose [1] in order to unify the quantum mechanical and the spacetime descriptions of Nature. It is well known that quantum mechanics deals with mathematical methods based on the complex structure of a Hilbert space of physical states (the probability amplitudes are the complex numbers). On the other hand, the theory of relativity demands the spacetime points to be described by real fourvectors (the coordinates of the spacetime events are the real numbers). The difficulties in a consistent formulation of a relativistic quantum theory are immediately related to this fact.

The main idea of the twistor theory is to treat the real coordinates of spacetime points as composed quantities of the complex objects so called twistors. Therefore, in the twistor theory the most fundamental objects are the twistors instead of the real spacetime points.

Mathematically twistors are the conformal O(4,2) spinors i.e. the complex fourvector in the fundamental representation of a covering conformal group SU(2,2) ≈ $\overline{O(4,2)}$. A correspondence between the twistors and the spacetime points is given by the incidence equation - Penrose relation.

The twistor formalism formulated originally by Penrose for the four - dimensional (D=4) spacetime can be extended in two ways:

i) extending the Penrose-relation in a supersymmetric way one obtains a correspondence between the supertwistors and the points of D=4 superspaces [2,3],

ii) replacing the complex numbers by quaternios in the Penrose relation one can bring the quaternionic twistors into connection with thepoints of the D=6 spacetime [4]. Furtheron, one can extend this quaternionic twistor formalism supersymmetrically introducing quaternionic fermionic degrees of freedom.

2. Composite D=4 spacetime from twistors.

Let us consider the fundamental steps in a more didactical way leading to the formulation of the Penrose-relation.

It is well known that any spacetime point described by the fourvector $x=(x^0,x^1,x^2,x^3)$ can be brought into connection with a hermitean 2x2 dimensional matrix, using the Pauli matrices σ_μ:

$$x \longleftrightarrow X = \begin{pmatrix} x^0+x^3 & x^1-ix^2 \\ x^1+ix^2 & x^0-x^3 \end{pmatrix} = x^\mu\sigma_\mu \qquad (1)$$

this correspondence is one to one.

One can also consider the complex fourvector $z=(z^0,z^1,z^2,z^3)$ instead of the real one x. The complex fourvector z describes a point of the complexified Minkowski Space $\mathbb{C}M^4$. A similar relation to (1) gives us the correspondence between the points of $\mathbb{C}M^4$ and two dimensional complex matrices:

$$z \longleftrightarrow Z = z^{\mu}\sigma_{\mu} \qquad (2)$$

One can get to the real Minkowski space $\mathbb{RM}^4$ by putting the reality condition onto the complex matrix Z i.e.

$$z \longrightarrow \quad \text{if} \quad Z = Z^{+} \qquad (3)$$

where Z^{+} denotes a hermitean conjugated matrix.

A point in the twistor construction is the use of isomorphism between complex two dimensional matrices Z and Z-plane in a fourdimensional complex vector space $\mathbb{C}^4$- the twistor space $\mathbb{T}=\mathbb{C}^4$. This isomporphism is given by the following correspondence [5]:

$$Z \longrightarrow \left\{\text{subspace spanned by columns of } 4\times2 \text{ matrix } \begin{bmatrix} iZ \\ I_2 \end{bmatrix}\right\} \qquad (4)$$

or more explicitly, the 4×2 matrix columns are identified with two twistors $T_1, T_2 \in \mathbb{T}$:

$$\begin{bmatrix} i\ Z \\ I_2 \end{bmatrix} = \begin{bmatrix} iz^0+iz^3 & z^2+iz^1 \\ iz^1-z^2 & iz^0-iz^3 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = (T_1, T_2) \qquad (4a)$$

From a mathematical point of view the correspondence (4) gives an affine system of coordinates for the Z-plane in the twistor space $\mathbb{T}$. This subspace is a complex Grassmann manifold $G_{2,4}(\mathbb{C})$. In other words, the Z-plane is given by the two linearly independent twistors $T_1, T_2 \in \mathbb{T}$.

Therefore, the relation (4) gives us the correspondence between the complexified spacetime point $z \in \mathbb{CM}^4$ and a complex Z-plane in the twistor space $\mathbb{T}$.

On the other hand, there is not a unique relation between the pair of twistors (T_1, T_2) and the Z-plane generated by this pair. It is clear, that every pair of twistors (T_1', T_2') is related to a nonsingular 2×2 matrix as follows.

$$(T_1', T_2') = (T_1, T_2)M \qquad (5)$$

gives the same Z-plane in the twistor space $\mathbb{T}$.

Let the pair (T_1', T_2') has the form (4a), therefore any equivalent pair of twistors satisfy

$$\begin{bmatrix} i\dot{Z} \\ I_2 \end{bmatrix} = (T_1, T_2)M = \begin{bmatrix} \Omega M \\ \Pi M \end{bmatrix} \Longleftrightarrow \begin{matrix} i\dot{Z} = \Omega M \\ I_2 = \Pi M \end{matrix} \qquad (6a)$$

where the 2×2 complex matrices Ω, Π are constructed of the coordinates of the twistors T_1, T_2.

Therefore, we obtain

$$iZ = \Omega\Pi^{-1} \Longleftrightarrow \Omega = iZ\Pi \qquad (6b)$$

This is a Penrose relation in matrix form.

Let us denote

$$(T_1, T_2) = \begin{bmatrix} \omega^{11} & \omega^{12} \\ \omega^{21} & \omega^{22} \\ \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \Omega \\ \Pi \end{bmatrix}, \qquad (7a)$$

now, from (6b) we obtain

$$\begin{matrix} \omega^{\dot{\alpha}1} = iz^{\dot{\alpha}\beta}\pi_{\beta 1} \\ \omega^{\dot{\alpha}2} = iz^{\dot{\alpha}\beta}\pi_{\beta 2} \end{matrix} \qquad \alpha, \beta = 1,2 \qquad (7b)$$

or more simply

$$\omega^{\dot{\alpha}} = iZ^{\dot{\alpha}\beta}\pi_{\beta} \qquad T = \begin{bmatrix} \omega^{\dot{\alpha}} \\ \pi_{\beta} \end{bmatrix} \qquad (7c)$$

it is the incidence equation postulated first by Penrose.

Its physical meaning is the following [1]:

the point $z \in \mathbb{CM}^4$ corresponds to the twistor $T \Longleftrightarrow \omega^{\dot{\alpha}} = iz^{\dot{\alpha}\beta}\pi_{\beta}$

It is obvious that all twistors lying on the Z-plane given in the (4) relations correspond to a given $z \in \mathbb{CM}^4$ point and for a given twistor T satisfying (7c) only one complex spacetime point z is assigned.

If one needs to describe the real space-time point $x \in \mathbb{RM}^4$, one should require the matrix Z to be hermitean i.e.

$$Z = Z^+ \Rightarrow Z = -i\Omega\Pi^{-1} = i(\Pi^{-1})^+\Omega^+ \qquad (8a)$$

therefore we get the following reality condition:

$$\Pi^{+}\Omega + \Omega^{+}\Pi = 0 \qquad (8b)$$

or using the notation (7a) we have three relations:

$$\pi^{*}_{\dot{\alpha}1}\,\omega^{\dot{\alpha}1} + \omega^{*\alpha 1}\pi_{\alpha 1} = 0$$
$$\pi^{*}_{\dot{\alpha}2}\,\omega^{\dot{\alpha}2} + \omega^{*\alpha 1}\pi_{\alpha 2} = 0 \qquad (8c)$$
$$\pi^{*}_{\dot{\alpha}1}\,\omega^{\dot{\alpha}2} + \omega^{*\alpha 2}\pi_{\alpha 1} = 0$$

where $\pi^{*}_{\dot{\alpha}\beta} = (\pi_{\alpha\beta})^{*}$, $\omega^{*\alpha\beta} = (\omega^{\dot{\alpha}\beta})^{*}$ and $*$ denotes the complex conjugation.

In the twistor framework the equations (8c) say that the twistors T_1, T_2 are "null-twistors" with respect to the U(2,2) norm:

$$(T_1,T_2) = (T_1,T_1) = (T_2,T_2) = 0 \qquad (9)$$

where

$$(T,T) = T^{+}GT = \begin{pmatrix}\omega^{*\alpha} & \Pi^{*}_{\dot{\beta}}\end{pmatrix}\begin{bmatrix}0 & I_2\\ I_2 & 0\end{bmatrix}\begin{pmatrix}\omega^{\dot{\alpha}}\\ \Pi_{\beta}\end{pmatrix}$$

and

$$I_2 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Therefore, the reality condition is equivalent to the zero condition for twistors i.e. to vanishing the U(2,2) norm of twistors T. The Z-plains generated by the "null twistors" are called totally null planes.

In this way we obtain the following correspondence diagram:

complex planes in $\mathbb{T}$	$\longleftrightarrow$	points of $\mathbb{C}M^4$ (complex Minkowski space)
$\downarrow$		$\downarrow$
totally null planes in $\mathbb{T}$	$\longleftrightarrow$	points of $\mathbb{R}M^4$ (real Minkowski space)

We would like to stress here, that from the point of view of the twistor theory, regaroling the relation (7c), it is more natural to use twistors for the description of the complex

Minkowski Space or the null twistors for that of the real Minkowski space time.

3. Supersymmetric extension of the Penrose incidence equation.

The aim of supersymmetry is to give a unified mathematical description of bosonic and fermionic fields. Therefore, one can consider bosons and fermions using the same theoretical scheme. Supersymmetry allows us to transform the descriptions of bosonic fields into fermionic ones and vice versa. (For more interested reader in this subject we recommend the references [6]). Therefore, in order to have a possibility of the description of bosonic and fermionic fields by using the twistor theory one has to extend it supersymmetrically.

The supersymmetry replaces the notation of a space-time point $x=(x^0,x^1,x^2,x^3)$ by an appropriate $\chi=(x^0,x^1,x^2,x^3;\ \theta_1,\theta_2,\ldots,\ \theta_N)$ point of the superspace adding N Grassmann variables $\theta_1,\ldots,\theta_N$. These additional degrees of freedom anticommute themselves.

Now, we can define a supervector representing D=4 N-extended superspace as follows

$$\chi = (x^0,x^1,x^2,x^3;\ \theta_1,\ldots,\theta_N) = (x^\mu;\theta_A) \qquad (11a)$$

where

$$\begin{aligned} &\mu = 0,\ \ldots,\ 4 \quad ; \qquad A = 1,\ \ldots,\ N \\ &[x^\mu,x^\nu] = x^\mu x^\nu - x^\nu x^\mu = 0 \\ &\{\theta_A,\theta_B\} = \theta_A\theta_B + \theta_B\theta_A = 0 \\ &[x^\mu,\theta_A] = x^\mu\theta_A - \theta_A x^\mu = 0 \end{aligned} \qquad (11b)$$

The commuting coordinates of a supervector are called bosonic ones whereas its anticommuting coordinates are called anticommuting ones.

In the same spirit one can generalize the twistor approach introducing N-extended supertwistors $T^{(n)}=(\omega^\alpha,\Pi_{\dot\beta};\xi_1,\ldots,\xi_N) \in \mathbb{C}^{4;N}$ (bosonic supertwistors) and the fermionic N-extended supertwistors

$\tilde{T}^{(N)} = (\eta_1,\ldots,\eta_4;u_1,\ldots,u_N) \in \mathbb{C}^{N;4}$, where the η_i quantities are fermionic coordinates and the u_A quantities are the bosonic ones [3].

Let us discuss the case of N=1 i.e. that of the simple supersymmetry briefly.

(i) Two linearly independent supertwistors $T_1^{(1)}, T_2^{(1)}$ span (2;0) - superplane in the superspace $\mathbb{C}^{4;1}$, in analogy to eqs.(6a,b) we get

$$(T_1^{(1)}, T_2^{(1)}) = \begin{bmatrix} \omega^{\dot{1}1} & \omega^{\dot{1}2} \\ \omega^{\dot{2}1} & \omega^{\dot{2}2} \\ \xi^1 & \xi^2 \\ \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} iZ & \\ \theta^1 & \theta^2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \Pi \qquad (12)$$

where Z and Π are complex matrices of 2x2 type made up of bosonic elements. This can be expressed (cf.eqs.(7)) as follows

$$\begin{aligned} \omega^{\dot{\alpha}1} &= iz^{\dot{\alpha}\beta}\pi_{\beta 1} \\ \omega^{\dot{\alpha}2} &= iz^{\dot{\alpha}\beta}\pi_{\beta 2} \\ \xi^1 &= \theta^1\pi_{11} + \theta^2\pi_{21} \\ \xi^2 &= \theta^1\pi_{12} + \theta^2\pi_{22} \end{aligned} \qquad (12a)$$

Therefore we obtain the sypersymmetric extension of the Penrose relation (7c) in the form

$$\begin{aligned} \omega^{\dot{\alpha}} &= iz^{\dot{\alpha}\beta}\pi_\beta \\ \xi &= \theta^\alpha\pi_\alpha \end{aligned} \qquad (12b)$$

It means that each $T^{(1)}=(\omega^{\dot{\alpha}},\pi_\beta,\xi)$ supertwistor corresponds to a (z,θ^α) superspace point.

However, it is not the only one possibility of the supersymmetrical generalization of the Penrose relation (7c).

ii) Applying three linearly independent supertwistors $T_1^{(1)}, T_2^{(1)}, \hat{T}^{(1)}$ (two bosonic and one fermionic space $\mathbb{C}^{4;1}$.

In analogy to (12) we have

$$(T_1^{(1)}, T_2^{(1)}, \tilde{T}^{(1)}) = \begin{bmatrix} \omega^{11} & \omega^{12} & \rho^{1} \\ \omega^{21} & \omega^{22} & \rho^{2} \\ \pi_{11} & \pi_{12} & \eta_1 \\ \pi_{11} & \pi_{12} & \eta_2 \\ \xi^1 & \xi^2 & u \end{bmatrix} = \begin{bmatrix} iz^{11} & iz^{12} & \theta^1 \\ iz^{21} & iz^{22} & \theta^2 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{11} & \pi_{12} & \eta_1 \\ \pi_{21} & \pi_{22} & \eta_2 \\ \xi^1 & \xi^2 & u \end{bmatrix} \qquad (13)$$

where the fermionic supertwistor includes the four fermionic $(\rho^{\dot{1}}, \rho^{\dot{2}}, \eta_1, \eta_2)$ components and also one bosonic u.
The (2;1) - superplane is parametrized by a (Z, θ) matrix of 2×3 type with elements satisfying the following incidence relations:

$$\omega^{\dot{\alpha}} = iz^{\dot{\alpha}\beta}\pi_b + \theta^{\dot{\alpha}}\xi \qquad \text{(bosonic incidence equation)} \qquad (14a)$$

$$\rho^{\dot{\alpha}} = iz^{\dot{\alpha}\beta}\eta_b + \theta^{\dot{\alpha}}u \qquad \text{(fermionic incidence equation)} \qquad (14b)$$

These equations give us a different generalization of the Penrose relation from (12b).

Therefore, for N=1 supersymmetry there are two possible extensions of Penrose's relation. In case of the N-extended supersymmetry one can genaralize the equation (7c) in N+1 different ways. The case of arbitrary N is considered in ref. [3].

4. Quaternionic extension of Penrose's incidence equation for D=6 spacetime.

There are two possible approches to D=6 twistor formalism:

(i) by extending Penrose's relation from D=4 to D=6 as it has been done by Hungston and Shaw in ref. [4].

(ii) by replacement of the complex 2×2 matrix $\mathbb{Z}$. In this approach the quaternionic 2×2 matrix $\mathbb{Z}=\mathbb{Z}^+=\mathbb{X}$ describes sixdimensional (D=6) real Minkowski spacetime point. One can show that these two approaches are equivalent for the

description of the real sixdimensional spacetime $\mathbb{R}M^6$.
First let us discuss first the case (i)

One can consider the complex D=6 twistors: $T=(\omega^a,\pi_a) \in \mathbb{C}^8$ $(a=1,\dots,4)$ as the norm of the spinors for eight dimensional complex orthogonal group $O(8,\mathbb{C})$ is:

$$(T,T') = \omega^a,\pi'_a + \pi_a\omega'^a = 0 \qquad (15)$$

the points of the complex D=6 Minkowski space $\mathbb{C}M^6$ are represented by a complex 4×4 antisymmetric matrix $z^{ab}= -z^{ba}$. The Penrose-incidence equation takes the form

$$\omega^a = z^{ab}\pi_b \qquad a,b = 1,\dots,4 \qquad (16)$$

This equation has a nontrivial solution if the twistors T are pure (simple) i.e.

$$(T,T) = 2\omega^a\pi_a \qquad (17)$$

in other words they have vanishing $O(8;\mathbb{C})$ norm.

The points of the real six dimensional Minkowski space $\mathbb{R}M^6$ are represented by a 4×4 complex, antisymmetric matrix Z satisfying the reality condition in the form

$$Z = -Z^* \quad \text{where} \quad Z^* = B^{-1}Z^+B , \qquad B = \begin{bmatrix} 0 & 1 & & \\ -1 & 0 & & \\ & & 0 & 1 \\ & & -1 & 0 \end{bmatrix} \qquad (18)$$

and Z^+ denotes the hermitean conjugated matrix. This reality condition for matrix Z is equivalent to the following condition for twistors

$$\omega^{*a}\pi_a + \pi^*_a\omega^a = 0 \qquad \text{where} \qquad \begin{aligned} \omega^{*a} &= \omega^{*b}(B^{-1})_b{}^a \\ \pi^*_a &= \pi^*_b(B)^b{}_a \end{aligned} \qquad (19)$$

and * means the complex conjugation.
The equation (19) is in fact the condition of vanishing the V(4,4) norm. Therefore, D=6 twistors describe the points of the real Minkowski space $\mathbb{R}M^6$ if the following two norms are zero:

$$O(8;\mathbb{C}) - \text{norm:} \qquad \omega^a\pi_a = 0 \qquad (20a)$$

$$U(4,4) - \text{norm:} \qquad \omega^{*a}\pi_a + \pi^*_a\omega^a = 0 \qquad (20b)$$

It means that D=6 twistors describing the points of $\mathbb{R}M^6$ are invariant under quaternionic orthogonal group $O(4;\mathbb{H})$ covering the conformal six dimensional group $O(6,2)$:

$$O(4;\mathbb{H}) \equiv U_{\alpha}(4;\mathbb{H}) = O(8;\mathbb{C}) \cap U(4,4) = \overline{O(6,2)} \qquad (21)$$

for details see ref: [7].

Therefore one can look for the quaternionic extension of D=4 twistor formalism which can describe $\mathbb{RM}^6$ Minkowski space.

Now, let us consider the case (ii).

First, we recall some basic properties of the quaternions $\mathbb{H}$, and recommend the references [8] on this subject.

The quaternions $\mathbb{H}$ constitute a four-dimensional real associative algebra with identity $1 \equiv e_0$. Any quaternion q is given by the sum:

$$q = q_0 e_0 + q_1 e_1 + q_2 e_2 + q_3 e_3 \qquad q_\mu \in \mathbb{R}, \quad \mu=0,1,2,3 \qquad (22)$$

where the quaternionic units satisfy the following multiplication rule:

$$e_i e_j = -\delta_{ij} + \epsilon_{ijk} e_k \qquad i,j,k = 1,2,3 \qquad (23)$$

Let us notice that the real numbers $\mathbb{R}$ are naturally embedded in $\mathbb{H}$ by identifying $q_0 e_0 = q_0 \in \mathbb{R}$.

For quaternions one can define a quaternionic conjugation (so called principal involution) writting

$$\bar{q} = q_0 - q_1 e_1 - q_2 e_2 - q_3 e_3 \qquad (24a)$$

and the norm

$$|q|^2 = q\bar{q} = q_0^2 + q_1^2 + q_3^2 \qquad (24b)$$

Therefore, the algebra $\mathbb{H}$ has the natural structure of the four-dimensional Euclidean space.

Sometimes it is useful to identify a quaternion q with the ordered pair of complex numbers (z_1, z_2) by

$$q = z_1 + e_2 z_2 = (q_0 + q_3 e_3) + e_2(q_2 + q_1 e_3) \qquad (25)$$

We can see that quaternions are the natural extensions of the real numbers $\mathbb{R}$ as well as complex ones $\mathbb{C}$.

Now, in analogy tp (4) for the given 2×2 quaternionic matrix $\mathbb{Z}$ we can associate Z-plane in fourdimensional quaternionic space $\mathbb{H}^4$ -

quaternionic twistor space as follows

$$\mathbb{Z} \longrightarrow \left\{ \text{subspace space by columns of } 4\times 2 \text{ quaternionic matrix } \begin{bmatrix} e_2 \mathbb{Z} \\ I_2 \end{bmatrix} \right\} \tag{26}$$

By a similar procedure to eqs.(4,5,6) we get the quaternionic Penrose-relation

$$\omega^{\alpha} = e_2 \dot{\mathbb{Z}}^{\alpha\beta}\ {}_{\beta} \qquad \alpha,\beta = 1,2 \tag{27}$$

where the quaternionic twistor has the form $t = (\omega^{\alpha},\ \ {}_{\beta})$.

A real D=6 Minkowski spacetime point is described by a sixdimensional vector $x = (x_0, x_1, \ldots, x_5) \in \mathbb{R}M^6$ which can be mapped on a quaternionic Hermitean 2×2 matrix (cf.eq(1)):

$$x \longrightarrow \mathbb{X} = \begin{bmatrix} x_0 + x_5 & x_4 + x_k e_k \\ x_4 - x_k e_k & x_0 - x_5 \end{bmatrix} \qquad k = 1,2,3 \tag{28}$$

The reality condition $\mathbb{Z} = \mathbb{Z}^{+}$ ($\mathbb{Z}^{+}$ denotes a quaternionic conjugated and transposed matrix) is equivalent to the following condition for quaternionic twistor t

$$\langle t,t \rangle = \bar{\omega}^{\alpha} e_2 \ \ {}_{\alpha} + \bar{\ }_{\alpha} e_2 \omega^{\alpha} = 0 \tag{29}$$

therefore, twistors t describe a point of $\mathbb{R}M^6$ if their $O(4;\mathbb{H}) \equiv$ $\equiv U_{\alpha}(4;\mathbb{H})$ norms vanish.

Using the decomposition (25) of quaternionic coordinates of twistor $\mathbb{H}$ one can immediately show that eq. (24) is equivalent to the relations (20), so the descriptions of $\mathbb{R}M^6$ by the D=6 complex twistors and D=6 quaternionic twistors are equivalent.

5. Final remarks.

It is worthwhile to notice that the two approaches above for the twistor description of D=6 spacetime are equivalent only for real space-time. This spacetime can be extended in two nonequivalent ways: by complexification or quaternionization procedures.

One can show also that the quaternionic formulation of twistor theory leads to serious difficulties with quantization of twistors because of the noncommutitavity of quaternions. However, the description of the D=6 spacetime in the quaternionic framework allows us to use the same geometry as in case of the complex description of D=4 spacetime.

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